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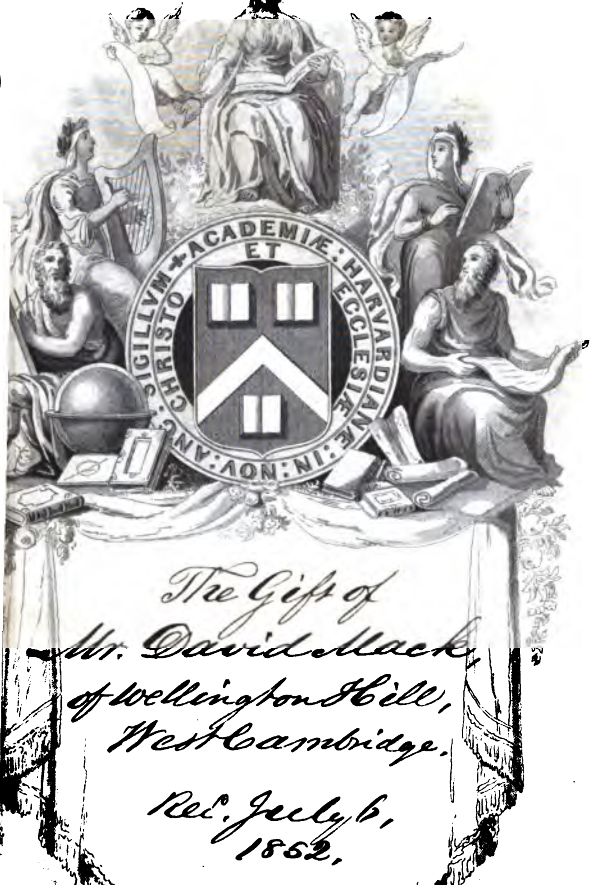
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The Gift of
Mr. David Mack,
of Wellington Hill,
West Cambridge.

Rec. July 6,
1852.

ARITHMETICK,

THEORETICALLY AND PRACTICALLY

ILLUSTRATED.

BY B. M. TYLER.

MIDDLETOWN, CONN.

PRINTED AND PUBLISHED BY E. & H. CLARK.

1827.

~~118.26.21~~

1852 July 6 Gift of
David Mack,
of Wellington Hill,
West Cambridge

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DISTRICT CLERK'S OFFICE.

BE IT REMEMBERED, That on the twenty-eighth day of November, A. D. 1826, in the fifty-first year of the Independence of the United States of America, B. M. TYLER, of the said District, has deposited in this Office the title of a Book, the right whereof he claims as Author, in the words following, to wit :

" *Arithmetick, Theoretically and Practically Illustrated. By B. M. Tyler.*"

In conformity to the Act of the Congress of the United States, entitled, " An Act for the encouragement of learning, by securing the copies of Maps, Charts, and Books, to the Authors and Proprietors of such copies during the times therein mentioned :" and also to an Act, entitled, " An Act, supplementary to an Act, entitled, An Act for the encouragement of learning, by securing the copies of Maps, Charts, and Books, to the Authors and Proprietors of such copies during the times therein mentioned; and extending the benefits thereof to the Arts of Designing, Engraving, and Etching Historical and other Prints."

CHARLES W. CUTLER,
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A true Copy as of Record : Attest,
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ARITHMETICK.

SECTION I.

DEFINITIONS.

A Definition is the import or meaning of a word, expressed by other words.

Quantity is any thing which can be increased, decreased or measured. [Number, lines, time, motion, space and matter, are quantities.]

Mathematicks is the science which treats of the numbering and measuring of quantities.

An Unit is one, or a single thing of any kind.

Number is an unit, or several units collected together. [One, two, three and ten, are numbers expressed by words.]

Arithmetick is that part of Mathematicks which treats of numbers; or, it is the science of numbering.

A Representative is a word, or character, standing for some other thing. [1 is a representative of a single thing of any kind.]

EXPLANATION OF ARITHMETICAL CHARACTERS OR FIGURES.

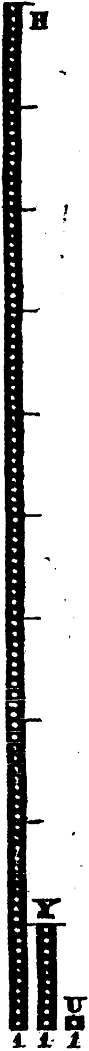
Numbers are generally expressed in Arithmetick by the ten following characters or figures;—1, 2, 3, 4, 5, 6, 7, 8, 9, 0*. Each of these figures has its own particular name, by which it is always called when standing alone. These names are called the common names of the figures. The word written before each figure below, is its common name;—one 1, two 2, three 3, four 4, five 5, six 6, seven 7, eight 8, nine 9, cipher 0. Each figure, when standing alone, always denotes the same number, which is called its simple value. The unit, or 1, is the least figure used, and all the others, except the cipher, are made up of as many units as are expressed by their common name; thus, 1 represents one single thing,—2 represents, or stands for two ones, 3—three ones, 4—four ones, 5—five ones, 6—six ones, 7—seven ones, 8—eight ones, 9—nine ones. The cipher never represents any thing of itself. Its use will be explained hereafter.

When figures are written alone, they do not express quantity; for *this* they are dependent upon words or other characters; for if we write the figure 6, and the question be asked, six what? there is no definite answer to the question. When figures stand alone, they express a certain number of units which do not represent quantity, and they can only be said, in such situations, to be indefinite, or abstract numbers. But if we write a word or character expressing quantity in connection with the figure, the word or character denotes what particular kind of quantity is meant, and the figure expresses how much of that quantity is taken into calculation;—thus, in the expressions, 5 dollars, 5 hours, the words *dollars* and *hours* denote particular kind of quantity, and the figure 5 expresses the number of parts of each quantity considered; in this respect, figures represent real values.

From what has been said, it appears that parts of as many quantities can be expressed by the same figure, as we can find words to express different quantities, and that the real value of each unit in the figure, will vary in the same manner as the quantities themselves; for in the phrases, 2 dollars, 2 years, 2 acres, 2 men, the figure expresses the same number of parts of each quantity, but an unit in each place has a different value.

---

* These figures are called Arabic, because they were brought into Europe by the inhabitants of Arabia. Historians suppose they were brought into Arabia from India. They were first used in Europe *about the year* 1150.



COMBINATION OF FIGURES, OR THE METHOD OF WRITING TWO OR MORE FIGURES IN CONNECTION.

When several figures are written in connection, the right hand figure expresses the same number of units, and is called by the same name, as though it stood alone; but the figure in the second place expresses ten times as many units as it would if it stood alone, or at the right hand, and is called as many tens as it would express units if it stood by itself. The figure in the third place denotes as many hundreds as it would units if it stood alone, or as many tens as it would units if it stood in the second place. The figure in the fourth place signifies as many thousands as it would hundreds in the third place. If we write four 1's in connection, the 1 at the right hand is an unit; the 1 in the second place, is a collection of ten units represented by one figure; the 1 in the third place, a collection of a hundred units, and the 1 in the fourth place, a collection of a thousand units:

Thus,

thousand	hundred	ten	unit
1	1	1	1

In the adjoining plate, let us suppose the character under U to be a tube or hollow case, containing one cent resting upon its edge. Now, if we wish to represent this cent by a figure, the figure 1 standing directly under the character U, will express it. Next, let us suppose the tube under Y to be ten times as long as that under U, and filled with cents standing the same as the other, it would of course contain ten, and the figure 1, standing directly under it in the second place, represents the same number. Lastly, let us suppose the tube under H, to be ten times as long as that under Y, or made by placing ten other tubes, one above another, each equal to that under Y; it would hold ten times as many cents as the tube Y, or a hundred times as many as the tube U. But as a figure in the third place expresses one hundred units, the figure 1 standing under the tube H, represents as many cents as that tube would contain. Each unit in this number represents one cent; and whenever any other quantity is represented by figures, each unit will express one of the equal parts into which such a quantity is divided. To express the numbers between one and ten, we make use of the other figures, as 2, 3, 4, 5, &c. The names of all the figures standing in the second place, except the 1, are contractions; thus, 2 tens is called twenty,—3 tens, thirty,—4 tens, forty,—5 tens, fifty,—6 tens, sixty,—7 tens, seventy,—8 tens, eighty,—9 tens, ninety. The numbers between ten and twenty, twenty and thirty, &c. are expressed by placing a figure to the right hand of the tens. We express ten and one, ten and two, &c. by writing the figures 1, 2, 3, &c. to the right hand of the ten; thus, 11, 12, 13; and they may be read ten and one, ten and two, ten and three, and so on; but to express them in a shorter manner, we make use of the contractions, eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, and nineteen. To express the numbers between twenty and thirty, thirty and forty, &c. instead of saying two tens and one, two tens and two, three tens and one, three tens and two, &c. we repeat the contractions, twenty-one, *twenty-two*, thirty-one, thirty-two, &c. Any figure, when standing in any place to the left of the right hand place, represents ten times as many units, as it would do if it stood in the next place to the right. From this principle, by the use of the nine figures and the cipher, we can express an indefinite number of quantities. In the expression, 2 dollars, we express the number of dollars by the figure 2; but if we write 1 to the right hand of the 2, the 2 is removed into the second place, (21 dollars,) and denotes twenty dollars; and by writing another 1, (211 dollars,) to the right hand, the 2 is removed into the third place, and represents two hundred dollars. We might continue this operation at pleasure, and every time we place a figure to the right hand, those to the left would express ten times as many as they did before; so that it is easy to perceive, that we might express as many different quantities by the same figure, as we can place figures to the right hand. All the other figures increase in the same manner. The cipher, which is sometimes called zero, never has any value of itself, whether written alone or in connection with other figures; but when it is written with other figures, it alters *their* value. Thus, 1 denotes one, but if we write a cipher to the right hand of it, (10,) the 1 is removed into the second place, and signifies ten. But if the cipher be placed at the left hand of a figure, (01,) it does not alter the value of the figure, because it does not remove the figure from the right hand place. The value of a figure, when it does not stand at the right hand, is called its *local value.*

* * *

NUMERATION, OR READING FIGURES.

DEFINITIONS.

A Simple Name of numbers, is a word by which one place of figures in the same half period is distinguished;—units, tens, hundreds, are the simple names of figures.

A Compound Name, or Term, is a word by which three or six places of figures are called, when all, or a part of the places are supplied with figures;—thousands, millions, billions, trillions, &c.

A Period is six places of figures, reckoning from the right hand. Each period is divided into two parts, having three places of figures in each.

NOTE.—It should be observed that the word unit or units is used in several different senses. It is used to express the right hand place of figures, the three right hand figures of every period: it is also the name of the right hand period. But in all cases, except in the right hand period, it denotes more than simple ones; thus, when we say 1 million, our meaning is one in the period of millions, yet this 1 million implies ten hundred thousand simple units.

Reading figures is expressing by words, a number represented in figures. If one figure is to be expressed by a word, we have only to repeat its common name. If two figures are to be read, we first repeat the tens by their contractions and the common name of the right hand figure. When three places of figures are to be read, we commence at the left hand, call the figure in that place by its common name, then add the word hundred, and read the other two as before directed. The three left hand figures of every period express thousands, and are read in the same manner as the three right hand ones, with the addition of the word thousand. All the other periods are read the same as the right hand one, and then the name of the period is repeated.

After the pupil has learned to read the six right hand places of figures, or the first period, he has only to learn the names of the other periods to be able to read with ease any number of figures. The right hand period is called Units, the next Millions, then Billions, Trillions, Quatrillions, Quintillions, Sextillions, Septillions, Octillions, Nonillions, Decillions, Undecillions, Duodecillions. An unit of any period implies ten hundred thousand of the units in the *next* right hand period. The names of the periods are not names of different denominations, strictly speaking, because any figure in either of the periods denotes ten times as many units, as it would if it stood in the next right hand place, which is the peculiar arrangement of simple numbers.

REMARK. The periods and the subdivisions of the periods are now generally divided by a comma. This practice is objectionable; for when the three right hand figures are decimals, as decimals are separated from whole numbers by commas, the scholar generally labours under a difficulty, to determine whether they are whole numbers or decimals, and in any case they tend to confuse his conceptions of the subject. In this work, the periods and half periods are separated by apostrophes, and decimals by commas. The method of reading figures is exhibited in a condensed form in the following table. The words at the right hand express the same numbers as the figures standing against them.

Hundreds of Millions,	Tens of Millions,	Millions,		Hundreds of Thousands,	Tens of Thousands,	Thousands,		Hundreds,	Tens,	Units,	
										1	One.
									1	2	Twelve.
								1	2	3	One hundred and twenty-three.
						1	'	2	3	4	One thousand 234.
					1	2	'	3	4	5	Twelve thousand 345.
				1	2	3	'	4	5	6	One hund. 23 thous. 456.
		1	'	2	3	4	'	5	6	7	1 million 234 thous. 567.
	1	2	'	3	4	5	'	6	7	8	12 millions 345 thous. 678
1	2	3	'	4	5	6	'	7	8	9	One hundred twenty-three million four hundred fifty-six thousand seven hundred eighty-nine.

When any place is supplied by a cipher, the cipher is not mentioned, and the other figures are read as before directed.

Hundreds of Millions,	Tens of Millions,	Millions,		Hundreds of Thousands,	Tens of Thousands,	Thousands,		Hundreds,	Tens,	Units,	
										2	Two.
									2	0	Twenty.
								3	0	2	Three hundred and two.
						4	’	0	0	3	Four thousand and three.
					5	0	’	0	4	0	Fifty thousand and forty.
				6	0	5	’	0	0	0	Six hundred five thousand
		7	’	0	0	0	’	0	0	0	Seven million.
	8	0	’	7	0	0	’	6	0	0	80 million 700 thous. 600.
9	0	8	’	7	0	6	’	5	0	4	Nine hundred and eight million 7 hundred and six thous. 5 hundred and four.

Let the scholar be required to read the following numbers.

76
765
87’654
76’543
222’222
999’999
20’000’000
10’000’010
66’065’064
100’000’000’000
404’404’303’303
742’921’532’119

Period of Quatrillions.		Period of Trillions.		Period of Billions.		Period of Millions.		Period of Units.	
908’	765’	432’	109’	876’	543’	210’	987’	654’	321
Thous. of Quatrill.	Units of Quatrill.	Thous. of Trillions.	Units of Trillions.	Thous. of Billions.	Units of Billions.	Thous. of Millions.	Units of Millions.	Thous. of Units.	Units.

The above five Periods of figures are read as follows :—nine hundred eight thousand, seven hundred sixty-five quatrillion, four hundred thirty-two thousand, one hundred and nine trillion, eight hundred seventy-six thousand, five hundred and forty-three billion, two hundred ten thousand, nine hundred eighty-seven million, six hundred fifty-four thousand, three hundred and twenty-one.

In reading the right hand period, the word *units* is not generally repeated, for it is always understood to be units unless the name of the period is mentioned.

◆

NOTATION.

(Notation is writing in figures any number expressed by words.) If a number of units less than ten, are to be written, we make use of one figure;—thus, nine is expressed by the figure 9. If more than nine units are to be written, we must set as many units in the second place as there are tens, because each one in the second place represents ten, and the other units at the right hand. Thus, twenty-two is written 22 because in twenty there are two tens; so, thirty-six is written 36; forty-eight, 48.

When three figures are to be written, place as many units in the third place as there are hundreds expressed, and the other two figures as before directed. If thousands are to be written, place the figures in the left division of the period, in the same manner as those in the right division. All the other periods are written in a similar manner.

Let the scholar be requested to write on his slate in figures, the following numbers.

Twenty-nine.

Five hundred and forty.

Six thousand nine hundred and seventy-two.

Eight thousand and six.

Seventy-four thousand and forty-one.

Six hundred, six thousand and six.

Forty-five million, four hundred nine thousand and eight.

Five hundred million, seven hundred one thousand and nineteen.

Thirty-five billion, fourteen million, seven hundred thousand and forty.

SECTION II.

DEFINITIONS.

A Question or Example is that which is required to be done.

A Rule is the direction by which a question or example is to be done.

An Answer is the result obtained by the correct performance of any question; or, it is the number demanded in the question.

Proof is the method of ascertaining whether a question has been correctly performed.

A Principle is the fundamental truth on which a rule is founded.

Illustration is the explanation of the principles or reasons on which any rule is established.

Addition is the collecting or putting several things or numbers together; thus, when we say 2 put with 1 makes 3, we add these two *numbers* together; so, if we take 2 dollars and put them with 3 others, we add two *quantities* together.

Subtraction is taking one number or one quantity from another; or it is taking a part of a number or quantity from a whole number or quantity. If we take 2 from 5, we take away 2 units which were contained in the 5, and there is 3 left; if we take 2 dollars from 5, we take away a part of the quantity; and in both cases we subtract.

Stating a Question is setting down numbers in a proper manner for the operation of a question.

Operation is the working or performing of a question.

REMARKS.

Addition and Subtraction are the bases of all operations in Arithmetick. This is evident from the following fact, that we cannot alter a number, unless we make it either larger or smaller. Neither can we alter a quantity, as it respects the amount of quantity, for Arithmetick considers quantity in no other respect, without making it more or less. But if we increase a number or a quantity we add to it; and if we lessen a number or quantity, we subtract from it.

◆

ADDITION OF SIMPLE NUMBERS.

A number is called simple when all the places of figures represent the same kind of quantity, or when ten in any one place is equal to one in the next left hand place; thus, 651 dollars, 723 years, are simple numbers. A given number is a number mentioned with which something is to be done.

The answer in addition is called the sum or amount.

The Addition of Simple Numbers is putting together two or more simple numbers, so that all of them may be expressed by one number.

Small numbers can be added in the mind. Thus, we say, 2 put with 4, make 6; 1 put with 7, make 8; 7 put with 3, make 10; 9 put with 4, make 13; 11 put with 5, make 16; 5 put with 13, make 18.

Let the pupil answer the following questions:

6 added to 3, make how many? 6 added to 2, make how many? 7 added to 2? 7 added to 4? 7 added to 5? 6 added to 7? 6 added to 9? 6 added to 4? 10 added to 1? 10 added to 4? 5 added to 10? 5 added to 11? 5 added to 12? 6 added to 12?

What is the sum of 6 and 14? of 7 and 14? of 8 and 13? of 3 and 14? of 2 and 15? of 4 and 16? of 15 and 5? of 7 and 14? of 6 and 15? of 8 and 16? of 7 and 17? of 9 and 15? of 19 and 5?

RULE

For stating Questions in Addition of Simple Numbers.

Set down the given numbers so that every figure may stand directly under one of its own local name; that is, units under units, tens under tens, hundreds under hundreds, &c.

State the following numbers for adding: 16'709, 87'634, 10'245, 1'175'302. They will stand thus,

```
      1 6 ' 7 0 9
      8 7 ' 6 3 4
      1 0 ' 2 4 5
1 ' 1 7 5 ' 3 0 2
_________________
_________________
```

RULE 1.

For adding simple numbers.

Add up each column by itself, commencing at the right hand, and set the sums underneath.

EXAMPLES.

```
Given     { 3 3 2 1
numbers,  { 2 1 3 3
          { 1 3 1 2
            -------
      Sum   6 7 6 6
```

Explanation of the operation.

In the first place, we addup the column of units, saying, 2 and 3 are 5, and 1 are 6. We then 1 add the column of tens, saying, and 3 are 4, and 2 are 6; next, the column of hundreds, 3 and 1 are 4, and 3 are 7; lastly, the column of thousands, 1 and 2 are 3, and 3 are 6.

(2.)

```
1 0 ' 2 3 1
3 2 ' 4 5 3
2 6 ' 1 1 4
-----------
6 8 ' 7 0 8
```

Explanation. As ciphers do not express any number, in adding we have nothing to do with them when they stand in a column with other figures.

(3.)	(4.)	(5.)
20'524	54'321	12'345
45'263	43'210	54'321
22'212	2'123	33'333
87'999	99'654	99'999

The pupil is required to state and work the following questions.

6. A man has two casks of grain; one contains 12 bushels, the other 21: How many bushels in both? Ans. 33.

7. A man bought a yoke of oxen for 56 dollars, and a horse for 43 dollars: How many dollars did he give for both? Ans. 99.

8. The eldest of three brothers has 102 cents: the second 75: and the third 22: How many cents have the three? Ans. 199.

9. A boy goes to a store and buys a yard of cloth for 45 cents: a book for 21: and a quart of molasses for 12: How many cents did he give for all the articles? Ans. 78.

10. What number of dollars are there in three bags: the first containing 1201 dollars: the second 3486: the third 2210?

Operation.

```
    1 2 0 1
    3 4 8 6
    2 2 1 0
    -------
Sum 6 8 9 7
```

Illustration. The sum of several numbers is equal to the numbers added together: for by adding up the several columns, we set a figure in the sum expressing as many units, tens or hundreds, &c. as there are units, tens or hundreds, &c. in all the given numbers. In the operation of the last question, 7 expresses the same number of units as 6 and 1 in the given numbers; the 9 tens, as many tens as there are tens in the given numbers; 8 hundred, as many hundreds as are represented by all the given numbers: and the 6 thousand denotes as many thousands as the thousands contained in the given numbers; therefore 6 thousand, 8 hundred, 9 tens and 7 units, or 6897, express a number equal to 1201, 3486 and 2210. If it were required to add the quantities which these given numbers express, we should put the dollars contained in the three bags into one bag, and the 6897 represents the number of dollars which the one bag would contain.

RULE II.

When the sum of any column is ten, or an exact number of tens, set a cipher under that column, and add one for each ten to the next column; but if the sum be more than an exact number of tens, set down what there is over the tens and add the tens as before.

(11.)

6 7 2
9 3 3
8 7 3
9 6 2

3 4 4 0

Explanation. In this example, because there is just ten in the right hand column, we set down a cipher, and add the 1 ten to the column of tens. The sum of the tens in the second column, with the one ten brought from the units' place, is 24. There being 2 tens in 20, we set down the 4 and add the two tens to the next column. The third column consists of 32, and the 2 brought from the tens' place make 34; we set down the 4, what there is over 3 tens, and as there is no other column, we set the 3 tens in the next place.

(12.)	(13.)	(14.)
6 7 6 9 1 2	7 0 0 1 2	9 0 0 0 0
1 2 4 5 3 6	1 2 3 4 5	8 7 0 0 0
7 6 8 9 0 2	9 8 7 6 5	7 6 5 0 0
1 2 3 4 5 6	8 7 6 5 4	6 5 4 3 0
		5 4 3 2 1
1 6 9 3 8 0 6	2 6 8 7 7 6	
		3 7 3 2 5 1

PROOF.

Begin at the top and add together all the columns of figures: and if the sums are the same as those produced when added from the bottom, the work is supposed to be correct.

Remark. No method of proving the operation of a question, will demonstrate the work to be correct, for the same error may be made in both operations; but there are methods of rendering the accuracy of all operations in Arithmetick more probable.

The scholar is required to prove the following questions.

(15.)	(16.)	(17.)
676912	20	6
76134	371	24
54887	7693	498
972	98564	6702
	839	17898

18. What is the sum of the following numbers; 67601, 87654, 1206 and 10101? Ans. 166562.

19. A certain farm is divided into 4 lots; the first containing 97 acres; the second, 108; the third, 70; and the fourth, 39. How many acres in the farm? Ans. 314.

20. What is the amount of twenty-five, two thousand five hundred, twenty-five thousand, and two hundred fifty thousand? Ans. 277525.

21. A merchant imported a bale of broadcloths containing 4 pieces, which measured as follows; the first, 31 yards; the second, 29; the third, 27; and the fourth, 32. How many yards of broadcloth were there in the bales. Ans. 119.

22. A butcher killed an ox, which weighed as follows; the two hind quarters, two hundred and eighty pounds; one of the fore quarters, one hundred and thirty-five pounds; the other, one hundred and forty-one pounds; the hide, eighty-nine; and the tallow, ninety; what was the whole weight of the ox? Ans. 735.

23. A gentleman owns 4 farms; the first is worth 965 dollars; the second, 3579; the third, 4018; *the fourth, 5617*; how much are they all worth?

Operation 1.	Operation 2.
965	965
3579	3579
4018	4018
5617	5617
29	14179
150	
2000	
12000	
14179	

Illustration. When the sum of the figures in any column, exceeds 9, we add the tens contained in that sum with the sum of the next column, because each 1 in the tens is equal to 10 in that column, and set the number above an exact number of tens, under the column last added; so that the sum of all the columns equal the given numbers; that is, the sum of all the columns contains as many units as all the given numbers. In the first operation of the last question, the sum of the right hand column is 29, and we place the 9 under that column, and set the 2 tens under the column of tens. The sum in the column of tens, is 15 tens, or 150; as 10 tens make 1 hundred, we place 1 for the 10 tens under the column of hundreds, and set 5, the remaining tens, under the column of tens. There being just 20, (or 20 hundred, which make 2 thousand,) in the column of hundreds, we have nothing to set in the place of hundreds, but carry the 2 tens of hundreds into the next place. The 2 in the 12 thousands, contained in the column of thousands, is set under that column and the ten, which is 10 thousand, is carried into the column of tens of thousands. By adding up these several sums, we obtain 14179 dollars, which is the total value of all the farms.

In the second operation, 9 is set under the right hand column, and the 2 tens added to the 5 tens, and 7, their sum, set down under the column of tens; the 10 tens being equal to 1 hundred, 1 hundred is added the next column. Lastly the 2 tens contained in column of hundreds, or 2 thousand, we add to 12, the sum of the thousands, and obtain the same answer as before.

24. Four men bought a piece of land in company; the first gave one hundred and seventeen dollars; the second, two hundred; the third, one hundred and sixty-two; and the fourth, one thousand and one; what did the land cost?

Ans. 1480 dollars.

25. Add together the following numbers; one thousand, six thousand and sixty, thirteen thousand and eight, four hundred and twenty.

Sum 20488.

26. How many feet of boards will it take to cover a building, if it require twenty-five hundred to board the sides; eighteen hundred and sixty-seven the ends; twenty-four hundred and thirteen feet to board the roof? Ans. 6780.

27. A gentleman paid four hundred and twenty-six dollars for lumber for the purpose of erecting a house; one hundred and three dollars for brick; thirty-five dollars for glass: forty-one for nails; for work, three hundred and fifteen; for boarding workmen, one hundred and eight; for lime, twenty-five; for the cellar, ninety-one; for paints, thirty-six. What was the cost of his house, if he pays one hundred and one dollars for other articles that have not been mentioned? Ans. 1281 dollars.

28. How many square miles on the surface of the earth, estimating Europe to contain 3387109; Asia, 16728002; Africa, 11652442; America, 16504254; and the islands, 4164420? Ans. 52436227.

This sign or character +, one line being horizontal, the other perpendicular, implies addition; or that the number before it is to be added to the number after it. It is sometimes called *plus*, because the word *plus* signifies *more*.

Two horizontal parallel lines = are used as a sign of equality, implying that the numbers before it are equal to the numbers after it. It is generally read *is*, or *are*, *equal to*;

Thus, $6+9=15$; or 6 plus $9=15$, and the expression is read, 6 added to 9 are equal to 15, or 6 plus 9 equals 15.

The scholar is required to work the following examples as directed by the signs:

29. 6724+11971=18695.
30. 269+109+210=588.
31. 245690+44444+555=290689.
32. 101+1010+10101+900830=912042.
33. 66 plus 83=149.
34. 400 plus 527 plus 601=1528.
35. 70000 plus 8008 plus 606 plus 55=78669.
36. 9 plus 99 plus 888 plus 7777 plus 66666=75439.

37. Seventeen thousand, one hundred and six+six hundred thousand, nine hundred and one+five million, five hundred and five thousand, five hundred and fifty=6123557.

38. A gentleman gave his estate to his three sons in the following order: to the youngest, four thousand and fifty dollars; to the second, one thousand and seventy-five dollars more than to the third; and to the first, as much as to the second and third; what did the eldest son receive, and what was the amount of the estate.

Ans. The eldest son received 9175 dollars. The whole estate was 18350 dollars.

Explanation.—The last answer is found by adding the first to itself.

39. From the creation of the world to the flood was 1656 years; from the flood to the destruction of Sodom and Gomorrah, 451 years; from the destruction of Sodom and Gomorrah to the Christian Era, 1897 years;—how many years from the creation of the world to the Christian Era? also, how many years from the creation of the world to the year 1826 of the Christian Era?

Ans. To the Christian Era, four thousand and four years. To the year 1826 of the Christian Era, five thousand, eight hundred and thirty years.

40. A carpenter purchased shingles enough to lay *21 thousand upon* a house, 14 thousand upon a barn,

7 thousand upon a shed, and he had 11 thousand left; how many shingles did he purchase.

Ans. Fifty-three thousand.

41. A truckman hauls 1 hogshead of molasses, weighing 1026 pounds; a hogshead of rum, weighing 907 pounds; and 656 pounds of iron, at one load. What is the weight of the load?

Ans. 2589 pounds.

42. A butcher buys 4 droves of cattle numbering as follows: 1st. 106 head; 2d. 220; 3d. 85; 4th. 325; how many head of cattle does he purchase?

Ans. 736.

43. A gentleman's library contains 75 volumes upon Divinity; 119 upon Law; 13 on Medicine; 35 on History; and 100 on miscellaneous subjects? How many volumes in the library? Ans. 342.

44. A gentleman's yearly income is as follows: interest money, 60 dollars; compensation for services, 259 dollars; rent for a house, 73 dollars. What is his income? Ans. 392 dollars.

45. What number is equal to ten thousand, seven hundred and twenty, plus four thousand, five hundred and two, plus six hundred, twenty thousand and ninety, plus four hundred, four thousand, two hundred and one, plus one million, two thousand and seventeen?

46. What number is equal to nine thousand, nine hundred and nine, plus four hundred thousand and four, plus seven hundred, ten thousand and nineteen, plus seventy thousand, seven hundred and seventy, plus eight million, twenty-four thousand, two hundred and forty, plus seventy million, seven hundred, seven thousand and seven?

47. What number is equal to three hundred, three thousand, three hundred and thirty, plus four million and four, plus six million, six hundred thousand, seven hundred and seven, plus twenty-five million, twenty-five thousand and twenty-five, plus seven hundred and sixty-two million, ninety-four thousand and eighty-one, plus one hundred and one million, one hundred and ten thousand, one hundred and one?

The sum of the answers to the three last questions is 981096727.

48. From the birth to the crucifixion of Christ was 33 years; to the commencement of the reign of Constantine 273 years more; to the cessation of speaking Latin in Italy, 275; to the flight of Mahomet from Mecca to Medina, 41; to the invention of glass in England, 42; to the founding of Cambridge University, (England,) 251; to the invention of musical notes, 155; to the invention of the mariner's compass, 232; to the engraving or etching on copper, 158; to the first knowledge of Algebra in Europe, 34; to the first making of knives in England, 69; to the first British Parliament, 144; to the birth of Washington, 25; to the battle of Bunker's Hill, 43; to the Independence of the United States, 1 year; to proclaiming Washington first President of the United States, 13; to the battle of Waterloo, 26. In what year of the Christian Era was the

Battle of Waterloo?	Ans.	1815.
Independence of the U. S.?		1776.
Birth of Washington?		1732.
First British Parliament?		1707.
Flight of Mahomet?		622.

Explanation.—Each space of time in the last question, is the number of years between one event and that which next preceded it; consequently, by adding all the periods of time up to any particular event, we shall obtain the number of that year in which such an event took place.

* * *

MULTIPLICATION OF SIMPLE NUMBERS, OR A SHORT METHOD OF ADDING EQUAL NUMBERS.

When two or more unequal numbers are to be added together, the operation is performed by addition; but when two or more equal numbers are to be added, the operation is performed by Multiplication.

Multiplying is adding the same figure or number several times together in the mind. When we say 3 times 4 are 12, our meaning is, that 4 added together 3 times, make 12; or that 4 and 4 are 8 and 4 are 12. So when we say 4 times 6 are 24, the meaning is, that 6 added together 4 times, make the sum of 24. If it were required to add 3 and 4 together, we could not perform the operation by multiplying, for neither 2 times 3, nor 2 times 4 make 7, the sum of 3 and 4; in this case, we should be obliged to *add* 3 and 4 to obtain their sum.

But were it required to find the sum of two 3's, or two 4's, we could obtain their sum either by adding or multiplying; for we may say, 3 and 3 are 6, and 4 and 4 are 8, or two times 3 are 6, and 2 times 4 are 8; in both cases we obtain the same number, but in the former case, by *adding*, and in the latter by *multiplying*.

It may at first appear singular to the young scholar, that we are able to add large numbers together in the mind, without setting down the number to be added to itself but once; but his surprise will be removed by reflecting that we have but nine figures and a cipher to represent all numbers, so that by committing to memory the sums of the nine figures, added together any number of times not exceeding 9, we can with great ease, set down the sum of any number, added together any number of times, for we multiply by one figure only at a time.

DEFINITIONS.

A Multiplicand is a number which is to be added to itself several times, or the number to be multiplied.

A Multiplier is a number by which the multiplicand is multiplied.

A Product is a number found by multiplying the multiplicand by the multiplier, and is always equal to the number which would be obtained if the multiplicand were added to itself as many times as the multiplier contains units. [3 times 4 are 12 ; here

3 is the multiplier, 4 the multiplicand, and 12 the product.

A Multiplication Table is a table exhibiting the sum or product of any number from 1 to 12, added together, or multiplied any number of times not exceeding 12.

MULTIPLICATION TABLE.

2 times	1	are	2	5 times	1	are	5	8 times	1	are	8
	2		4		2		10		2		16
	3		6		3		15		3		24
	4		8		4		20		4		32
	5		10		5		25		5		40
	6		12		6		30		6		48
	7		14		7		35		7		56
	8		16		8		40		8		64
	9		18		9		45		9		72
	10		20		10		50		10		80
	11		22		11		55		11		88
	12		24		12		60		12		96
3 times	1	are	3	6 times	1	are	6	9 times	1	are	9
	2		6		2		12		2		18
	3		9		3		18		3		27
	4		12		4		24		4		36
	5		15		5		30		5		45
	6		18		6		36		6		54
	7		21		7		42		7		63
	8		24		8		48		8		72
	9		27		9		54		9		81
	10		30		10		60		10		90
	11		33		11		66		11		99
	12		36		12		72		12		108
4 times	1	are	4	7 times	1	are	7	10 times	1	are	10
	2		8		2		14		2		20
	3		12		3		21		3		30
	4		16		4		28		4		40
	5		20		5		35		5		50
	6		24		6		42		6		60
	7		28		7		49		7		70
	8		32		8		56		8		80
	9		36		9		63		9		90
	10		40		10		70		10		100
	11		44		11		77		11		110
	12		48		12		84		12		120

11 times 1 are	11	11 times 9 are	99	12 times 5	60
2	22	10	110	6	72
3	33	11	121	7	84
4	44	12	132	8	96
5	55	12 times 1 are	12	9	108
6	66	2	24	10	120
7	77	3	36	11	132
8	88	4	48	12	144

This Table should be perfectly committed to memory by every scholar before he attempts to proceed any further, as little progress can be made without it.

When the multiplier consists of one figure only.

RULE.

(Set the multiplier under the units' place of the multiplicand, and multiply every figure in the muliplicand by the multiplier, beginning at the right hand.)

Question 1. What is the product of 321, multiplied by 2?

Operation.

Multiplicand	3 2 1
Multiplier	2
Product	6 4 2

Explanation.—In this question, we say, twice 1 are 2, twice 2 are 4 and twice 3 are 6.

2. What is the product of 54321, multiplied by 2? Ans. 108642.

3. What is the product of 644312, multiplied by 2? Ans. 1288624.

4. Multiply 9321 by 3. Product. 27963.

5. Multiply 82122 by 4 Prod. 328488.

6. Multiply 7202 by 4.

Operation.

```
7 2 0 2
      4
-------
2 8 8 0 8
```

Explanation.—When there are ciphers in the multiplicand, as they do not represent any number of themselves, their products cannot be any thing but ciphers. The use of placing ciphers in the product, is to keep the product of the next figure in its proper place.

Illustration.—The product is the same number that the sum would be, if the multiplicand were added together as many times as the multiplier contains an unit. When we *add*, we set down the numbers, and add one column at a time; but when we *multiply*, we add together in the mind, each figure of the multiplicand, as many times as the multiplier contains an unit, which always shows how many numbers, each equal to the multiplicand, are to be added together; therefore, we shall obtain the same number by multiplying, as we should by adding the multiplicand together as many times as the multiplier contains an unit.

If it be required to multiply 432 by 3, we may obtain the answer by adding or multiplying.

Operation by adding.

```
   4 3 2
   4 3 2
   4 3 2
 -------
 1 2 9 6 Sum.
```

Operation by multiplying.

```
   4 3 2
       3
 -------
 1 2 9 6 Product.
```

In the first operation, we say, 2 and 2 are 4, and 2 are 6; 3 and 3 are 6, and 3 are 9; 4 and 4 are 8, and 4 are 12. But in the second operation, when we say, 3 times 2 are 6, our meaning is the same, as when we say 2 and 2 are 4, and 2 are 6; and 3 times 3 are 9, is the same as 3 and 3 are 6, and 3 are 9; and 3 times 4 are 12, the same as 4 and 4 are 8, and 4 are 12.

As the sum of each column in the addition, is equal to the product in the same place in the multiplication, the sum must equal the product.

It has been seen, that whether we multiply tens, hundreds, &c. the operations are the same as though we multiplied units; for 2 times 3 hundred make 6 hundred, and 3 times 4 thousand make 12 thousand.

Remark.—From what has been said, it is manifest, that when several things are sold at the same price for each one, the price of the whole may be found by multiplying; for if we set down the price of one article as many times as there are articles, and add up these prices, we shall obtain the price of the whole; but by multiplying the price of one article by the whole number, we obtain the same answer. From this it appears, when any thing is sold, and the price of one unit of the quantity is given, that the quantity will be the multiplier, and the price of an unit, the multiplicand.

7. A man sells 3 bushels of corn, at 32 cents a bushel; how many cents does he receive for the three bushels?

Operation by adding.	Operation by multiplying.
32	32
32	3
32	—
—	96
96	

Explanation.—In this question, the three bushels of corn is the quantity, and must be the multiplier; the 32 cents the price of one bushel, or an unit of the quantity, is the multiplicand. By setting down 32, the price of one bushel, 3 times, and adding, we obtain 96 cents, the price of 3 bushels in one number; but by multiplying 32 by 3, the number of bushels, we obtain the same answer. When the multiplier is a small number, we can readily obtain an answer by adding; but when the multiplier is a large number, it would be very tedious to set down the multiplicand as many times as there are units in the multiplier, and be obliged to add them up. This shows the great utility of multiplication.

8. What is the price of 4 barrels of flour, at 521 cents a barrel? Ans. 2084 cents.

9. A farmer has 5 fields of wheat. Each field produces 81 bushels; how many bushels in the 5 fields. Ans. 405.

10. A butcher gives 41 dollars a piece for 9 oxen; what does he give for them all?

Ans. 369 dollars.

NOTE 1.—When the product of two figures is more than 9, and an exact number of tens, set a cipher under the figure multiplied, and add as many ones to the next product, as there were tens; but if the product be not an exact number of tens, set down the right hand figure, and carry the tens as before.

11. What is the product of 9766, multiplied by 2?

Operation.
9 7 6 6
2
————
1 9 5 3 2

Illustration.—The reason for carrying the tens to the left hand product, is founded in the nature of simple numbers; for 1 in any column, except the right hand one, is equal to 10 in the column to the right hand of the same.

12. If the multiplicand be 16705, and the multiplier 6, what is the product?

Operation.
1 6 7 0 5
6
————
1 0 0 2 3 0

Explanation.—When we have tens to add to the next product, if the next be occupied by a cipher, the tens must be placed directly beneath it, as in this question.

13. Multiply ten thousand, six hundred and twenty-five by 8. Prod. 85'000.

14. Multiply one hundred and sixteen thousand, nine hundred and sixty-nine by seven.

Prod. 818'783.

15. A certain army was composed of nine battalions; each battalion consisted of six hundred and fifty-three men; how many men were there in the army? Ans. 5'877.

Two lines crossing each other thus ×, are used as a sign of Multiplication; and signify that the number before it is to be multiplied by the number after it; thus, 62×2=124; which is read 62 multiied by 2 is equal to 124. The sign of multiplica-n is frequently expressed by the word *into*; thus, *into 4=212*, and is read, 53 into 4 equals 212.

EXAMPLES.

16. 396×3=1188. 17. 2408×5=12040.
18. 7034 into 6=42204. 19. 1928 into 7=13496.
20. 6205×2 21. 15143 into 3
22. 23456×4 23. 34567 into 5
24. 45678×6 25. 56789 into 7
26. 67890×8 27. 78901 into 9

The pupil may ascertain whether he has worked the 8 last examples correctly, by adding up his products, the sum of which must equal 2'249'318.

28. A boy buys 15 lemons for 4 cents each; what does he give for them all? Ans. 60 cents.

29. A merchant sells a roll of riband, measuring 28 yards, for 8 cents a yard; what does he obtain for the roll? Ans. 224 cents.

30. What will 256 pounds of nails come to, at 6 cents a pound? Ans. 1536 cents.

Note 2.—100 cents make 1 dollar, therefore there will always be as many dollars in any number of cents as there are hundreds. Separate the two right hand figures from the others by a comma; those to the left will signify dollars, and those to the right, cents.

This sign $ is used to denote dollars. All the figures after the sign of dollars, to the comma, denote dollars, and those to the right of the comma, cents, if no more than two places; but if no comma is inserted, all the figures represent dollars.

31. A merchant sells 671 yards of tow cloth at 9 cents a yard; what is the price of the whole?
Ans. $60,39.

32. What will 6 hogsheads of rum come to, at 4555 cents a hogshead? Ans. $273,30.

33. What will 89 pounds of beef come to at 7 cents a pound? Ans. $6,23.

34. What will 101 boxes of sugar come to, at 9 dollars a box? Ans. $909.

When the multiplier consists of several figures.

RULE.

Place the multiplier under the multiplicand, units under units, tens under tens, hundreds under hundreds, &c. Multiply the multiplicand by each significant figure in the multiplier, beginning with the figure in units' place, and write the first figure of each product directly under the figure by which you multiply. Add the several products together, their sum will be the product of the multiplicand multiplied by all the figures in the multiplier.

35. What is the product of 26412, multiplied by 32?

Operation.

	2	6	4	1	2	
				3	2	
	5	2	8	2	4	product of 2.
7	9	2	3	6		product of 3 tens, or 30.
8	4	5	1	8	4	product of 32.

36. What is the product of 90821, multiplied by 432?

Operation.

			9	0	8	2	1	
					4	3	2	
		1	8	1	6	4	2	product of 2.
	2	7	2	4	6	3		product of 3 tens, or 30.
3	6	3	2	8	4			product of 4 hundred.
3	9	2	3	4	6	7	2	product of 432.

37. What is the product of 8765401, multiplied by 5432? Ans. 47'613'658'232.

38. Let the multiplicand be seven hundred sixty-five thousand, four hundred and two, and the multiplier 263; what is the product?

Ans. 201'300'726.

Illustration of the last rule.—The product of the unit figure in the multiplier, is found in the same manner as was shown in the illustration of the first *rule.* Each 1 in the place of tens, denotes 10 units, and the multiplicand must be brought into the product once for every single unit contained in any figure standing in tens' place. When we multiply the unit figure of the multiplicand by the tens in the multiplier, we make use of the same expressions as though the multiplier were in units' place; but by removing the product of the unit figure of the multiplicand into tens' place in the product, it signifies ten times as many as it would, if it stood in units' place. By removing the product of any other figure, when multiplied by the figure standing in tens' place in the multiplier, it denotes ten times as many as it would, if it stood directly under that figure of the multiplicand; consequently the product of any figure standing in tens' place in the multiplier, will express ten times as many as it would, if it stood one place further to the right hand. In multiplying by a figure standing in hundreds' place, we place the product of every figure in the multiplicand, two places further to the left, than we should if the figure by which we multiply, had stood in units' place; therefore the product must represent one hundred times as many as it would, if it were removed two places further to the right. The same might be shown of the product of a figure standing in any other place; therefore, by adding the several products together we obtain a number which is made up of the multiplicand, taken as many times as the multiplier contains an unit.

Let it be required to multiply 54321, *by* 321.

Operation.

$$\begin{array}{r}
5\,4\,3\,2\,1 \\
3\,2\,1 \\
\hline
5\,4\,3\,2\,1 \\
1\,0\,8\,6\,4\,2 \\
1\,6\,2\,9\,6\,3 \\
\hline
1\,7\,4\,3\,7\,0\,4\,1
\end{array}$$

When we multiply by the 2 in the multiplier, we say, twice 1 are 2; but as the 2 is 2 tens, our meaning is, that 20 times 1 are 20, and we obtain 20 by setting the 2 in tens' place. We also say, 2 times 2 are 4, but by setting the 4 in hundreds' place, the result is, that 20 times 20 are 4 hundred. In like manner, we say, 3 times 1 are 3, but the 3 is 3 hundred, and the product becomes the same by placing it in the third place.

The operation of this example may be performed by separating the multiplier into 3 parts, as follows:

5 4 3 2 1	5 4 3 2 1	5 4 3 2 1
1	2 0	3 0 0
5 4 3 2 1	1 0 8 6 4 2 0	1 6 2 9 6 3 0 0

1 6 2 9 6 3 0 0 is 3 hundred times 54321.
1 0 8 6 4 2 0 is 20 times 54321.
5 4 3 2 1 is once 54321.

1 7 4 3 7 0 4 1 the product 54321, multiplied by 321.

39. A gentleman has a field of corn consisting of 196 rows, each row has 219 hills. How many hills in the field? Ans. 42'924.

Explanation. The number of hills in each row must be taken as many times as there are rows.

40. What is the product of 612'813, multiplied by eleven thousand, one hundred and twelve? Ans. 6'809'578'056.

41. A merchant sells 309 bales of cotton at 41 dollars a bale; how many dollars does he receive for the whole? Ans. 12'669.

42. If one hundred and twenty-five men receive each one hundred and fifteen dollars for one year's labour, how much do they all receive. Ans. $14'375.

43. If a man spend 65 cents a day, what will be his expense for one year, it being 365 days? Ans. $237,25.

PROOF.

Multiply the multiplier by the multiplicand, and if the product be the same number as the product found by multiplying the multiplicand by the multiplier, the answer may be supposed correct.

Illustration. The foregoing method of proving multiplication, is founded on the following proposition, that the product of two numbers will always be the same number which way soever they are multiplied. If we multiply 9 by 6, or 6 by 9, we obtain the same result, viz. 54. To express the same by adding, let 9 be separated into two parts, and added together 6 times.

$6+3=9$
$6+3=9$
$6+3=9$
$6+3=9$
$6+3=9$
$6+3=9$

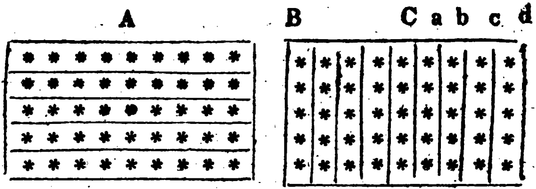
36 18 54
18

54

By adding the two left hand columns, we, in reality, add 6 nine times which is the same as though we had multiplied 6 by 9. When we add the right hand column, we do the same thing as when we multiplied 9 by 6; and we obtain the same number now as before.

To show that the products of any two figures, when multiplied both ways, will be equal, let us make 5 rows of points, each row consisting of nine points, as in the parallelogram A. Now if we set down the number of points in one row, 5 times, and add them, the sum 45 would be the number of points in all the rows; but if we multiply the number in one row by 5, the number of rows, we shall obtain the same number, for the product contains the multiplicand as many times as the multiplier contains an unit.

Next let us make 5 rows of points, each row consisting of 5, as seen in parallelogram B C; then suppose 4 points from each row in the parallelogram A, to be removed from the left and added to the right of the parallelogram B C; there will be points enough



to make the 4 rows a, b, c, d; then we have 9 rows, with 5 points in each, and by multiplying 5, the number of points in one row by 9, the number of rows, we shall obtain the number of points in the 9 rows, which is 45, the same number that we found in the parallelogram A, when we multiplied 9 by 5. We may suppose the parallelogram A to be increased by more rows, and each row to consist of more points, but by removing a part of them as before, and making rows the other way, we should always obtain the same product.

Let the scholar prove the following examples:

44. 9631×341. 45. 84213×919.
46. 705684×1763. 47. 123456×54321.
48. 9753201×98765. 49. 9897969×191817.

50. What is the product of one hundred sixty-seven thousand, five hundred and twenty-seven, into forty-five thousand, nine hundred and sixty-eight?

51. A merchant has a bale of bombazetts, consisting of 4 pieces, each piece 31 yards; and he gave 24 cents a yard. What did the 4 pieces cost him?
Ans. $29,76.

CONTRACTIONS.

The operations in multiplication may often be contracted, or shortened.

Contraction 1.—When ciphers stand at the right hand of the multiplier, set the multiplier so that the right hand significant figure may stand directly under units' place in the multiplicand. By bringing down the ciphers, the products of the other figures are brought into their proper places.

52. An army consisting of twenty-one thousand, three hundred and seventeen men, plundered so much money from a certain city, that each man received $50. How much money was taken?

Operation.

  21’317
      50

Ans. $1’065’850

Note 3.—By the illustration of the proof, it is immaterial which of the two numbers is made the multiplier; but it is generally more convenient to make the less number the multiplier.

53. 107821 × 11000. 54. 107821 × 990000.

The sum of the two last products is 107’928’821’-000.

55. An army has 150 waggons loaded with beef; each has six barrels, and each barrel contains 200 pounds. What is the whole number of pounds?
Ans. 180’000.

56. What number contains six thousand, seven hundred and sixty-five, nine thousand times?
Ans. 60’885’000.

57. If each window in a house, having 20 windows, contains 24 panes of glass, how many panes in all the windows? Ans. 480.

58. Suppose the number of inhabitants in the United States to be twelve million, and each person to be worth five hundred dollars; what are they all worth. Ans. 6’000’000’000 dolls.

Contraction 2.—If the multiplier be 1, with ciphers at the right hand, we have only to annex the ciphers in the multiplier to the right hand of the multiplicand, for multiplying by the 1 does not alter the figures in the multiplicand, and annexing the ciphers, removes every figure in the multiplicand into its proper place, in the same manner that it would, were we to go through the process of multiplying.

59. How many bushels of salt in 161 casks, each containing 10 bushels? Ans. 1'610.

60. One dollar contains 100 cents; how many cents is 65 dollars? Ans. 6500.

61. One thousand mills being equal to one dollar, how many mills in 65 dollars? Ans. 65'000.

62. If 12029 were set down ten thousand times, and added up, what would be the sum?
Ans. 120'290'000.

QUESTIONS TO BE WORKED BY THE ADDITION AND MULTIPLICATION OF SIMPLE NUMBERS.

1. Charles bought 29 oranges, and John 31, at 5 cents a piece. What did all the oranges cost?

Operation 1.	Operation 2.
29 31	29+31=60
5 5	5
145+155=$3,00	$3,00

Explanation.—The last question may be performed two ways, either by multiplying each quantity separately by 5, or adding the quantities together, and then multiplying. But had one boy given more a piece for his oranges than the other, the price of the whole would have been found as in the first operation, for 5 times 29, and 7 times 31 makes more than 300.

2. A man bought 101 bushels of rye for 45 cents a bushel; 65 pounds of tea for 45 cents a pound; what is the cost of the rye and tea?
Ans. $74,70.

3. A farmer sells 405 pounds of beef, at 6 cents a pound, and 5 tons of hay, at $15,00 a ton; what does he receive for both? Ans. $99,30.

4. What is the amount of the following articles,

viz.: 69 gallons of wine, at 81 cents a gallon; 59 pounds of sugar, at 13 cents a pound; 17 yards of riband, at 17 cents a yard; and 9 plates, at 7 cents a piece? Ans. $67,08.

5. The cost of constructing 25 miles of a certain road, is twenty thousand dollars; for the remaining part, which is 14 miles, the cost is fifteen thousand and twenty-four dollars a mile. What was the whole cost of the road? Ans. $230’336.

6. Multiply the sum of the following numbers by 203; viz. 1045, 69721, 621 and 10001.
Ans. $16’521’764.

7. A man gave his estate to 8 sons; to 5 of them he gave nine hundred and eleven dollars each; to the other 3, seven hundred and one dollars each; what was his estate? Ans. $6659.

8. Suppose 1002 men possess equal properties, and the property of each man to be as follows; 6000 acres of wild land, at 4 dollars an acre; buildings worth 4011 dollars; 999 sheep, at 2 dollars each; 3 horses, each 95 dollars; money at interest, 3 times the value of his buildings; 10 shares in a bank, 500 dollars each; and other property, the value of which is equal to the product of 25 into the value of his bank stock and wild land. What would be the property of all the men? Ans. $773’871’654.

Remark. When one man sells another several articles, they are generally set down upon a piece of paper; the price of each article found separately, and all the prices added up. If the buyer pays for the articles, the seller writes, *received payment*, at the bottom of the bill, and signs his name; but if payment is not made, the several prices are only added up.

BILLS OF PARCELS.

BOSTON, July 18, 1826.

Henry Fairdealer,

Bought of Thomas Richman,

35	yards of sheeting,	at	18	cents per yard.
9	“ “ cambrick,	“	53	“ “ “
11	“ “ linen,	“	42	“ “ “
3	“ “ muslin,	“	71	“ “ “
5	silk handkerchiefs	“	92	“ a piece,

Received Payment, $22,42.

Thomas Richman.

Note 4.—When the price of one unit of any quantity, is dollars and cents, we multiply it as a simple number, and point off the two right hand figures of the product, which will be cents, and those to the left, dollars. The operations in dollars and cents, will be illustrated in Decimal Fractions.

NEW-YORK, July 18, 1826.

John Clerk,

Bought of Charles Honesty,

9	yards of Broadcloth,	at	$7,33	per yard,
5	“ Cassimere,	“	1,75	“
11	“ Camblet,	“	,78	“
6	“ Holland,	“	,30	“
3	Waistcoat patterns,	“	1,42	

$89,36.

PHILADELPHIA, July 18, 1826.

Jonathan Buyer,

Bought of Benjamin Tradeall,

8	gallons Brandy,	at	$1,60	per gallon,
24	“ W. I. Rum,	at	,93	“
43	“ Gin,	“	,65	“
17	“ Molasses,	“	,45	“
56	pounds Brown Sugar	“	,14	per pound,
14	“ Loaf “	“	,23	“

Received payment by his note of the above date, $81,78

For Benjamin Tradeall,

Andrew Simson.

SECTION III.

SUBTRACTION OF SIMPLE NUMBERS.

Subtracting is taking one number from another; or taking a part from the whole.

In the preceding section, the principles of adding numbers together, have been explained; in *this*, we shall illustrate the methods of separating them. When two or more numbers are put together, so that they may all be expressed by one number or sum, we add numbers; but when we take away a part of a number, or take one number from another, we subtract numbers. When we say, 2 put with 4, makes 6, we add 2 to the 4; but when we say, 2 taken from 4, and 2 remains, we subtract the 2 from 4.

Subtraction is evidently the reverse of Addition.

DEFINITIONS.

A minuend is a number from which another number is to be taken; or, a number from which a part is to be taken, of course, it must always be as large or larger than the other numbers, for we cannot take from one number, another larger than itself.

A subtrahend is a number which is to be taken from another.

A remainder is a number which is left after one number is taken from another; or, it is the difference between two numbers. A remainder may equal, or be greater than the subtrahend, but never can equal the minuend. [2 taken from 5, leaves 3. Here 5 is the minuend, 2 the subtrahend, and 3 the remainder.]

Let the pupil answer the following questions:

What is the difference between 2 and 6? Between 5 and 7? Between 4 and 8? Between 1 and 8? Between 2 and 8? Between 1 and 9? Between 2 and 9? Between 4 and 9? Between 7 and

8? Between 3 and 8? Between 3 and 9? Between 3 and 10? Between 4 and 10? Between 5 and 10? Between 5 and 11? Between 6 and 11? Between 6 and 13? Between 13 and 7? Between 14 and 9? Between 15 and 11? Between 16 and 19? Between 17 and 21? How many more are 11 than 9? 19 than 15? 21 than 3? 29 than 24? 28 than 23? 31 than 29?

RULE 1.—State the question or example by setting the subtrahend, or less number, under the larger, so that every figure may stand directly under one of its own local name; that is, units under units, tens under tens, &c.

2. Commence subtracting at the right hand, and proceed regularly to the left, setting underneath the difference between every figure of the subtrahend and that which stands directly above it in the minuend.

Question 1. What is the difference between 68 and 45?

Operation.

Minuend, 6 8
Subtrahend, 4 5
———
Remainder, 2 3

Illustration.—In this question, 3 is the difference between the 5 and 8, and the 2, or 2 tens, is the difference between the 4 and the 6, or 40 and 60; therefore 20 and 3, or 23, is the difference between 68 and 45. The same answer may be obtained by separating the numbers.

Operation.

$60+8=68$
$40+5=45$
———
$20+3=23$

Explanation.—Here I separate the 8 from the 68, and the 5 from the 45. I then subtract the 5 from the 8, and the 40 from the 60, and add the remainders together, which gives the same answer as before.

2. Required the difference between 98 and 87?

Rem. 11.

3. How many more are 679 than 521?
Ans. 158.

4. George has 63 marbles, and James has 52; how many has George more than James?
Ans. 11.

5. A boy went to a store with seventy-nine cents, and spent thirty-eight of them; how many had he left?
Ans. 41.

6. Henry had 11 apples, and gave 8 of them to Thomas; how many had he left?
Ans. 3.

7. William said he had 35 cents; John said he had 24; how many more had William than John?
Ans. 11.

8. A merchant sells 137 pounds of sugar from a hogshead which contains 548 pounds; how many pounds remain?
Ans. 411.

9. A man gave 13 dollars to one son, and 37 to another; how many had one more than the other?
Ans. 24.

10. A man gave 5 dollars for a hat, and 19 dollars for a greatcoat; how much less did his hat cost him than his greatcoat?
Ans. $14.

11. What is the difference between six thousand, five hundred and thirty-nine, and four thousand, one hundred and eleven?
Ans. 2428.

12. One man has on hand 965 dollars, another, 623 dollars less; how many dollars has the latter?
Ans. $342.

13. Take 40 from 145, and what will remain?

Operation.

$$\begin{array}{r} 1\,4\,5 \\ 4\,0 \\ \hline 1\,0\,5 \end{array}$$

Explanation.—In this question, there being no units in the right hand place of the subtrahend, the 5 must be brought down. The number of tens in the less number, is equal to that in the larger; consequently, there is no difference, and the tens' place in the remainder must be supplied with a cipher. As there is no figure in hundreds' place in the less number, the hundred in the larger will not be lessened.

14. If you take 160 from 2375, how many will remain? Ans. 2215.

15. A merchant bought 1621 boxes of lemons; after selling eleven hundred, how many had he left? Ans. 521.

16. If a merchant buy twelve hundred and seventy-nine dollars worth of riband, and sell it for thirteen hundred and eighty-nine dollars, what does he gain? Ans. $110.

Note 1.—The gain must evidently be the difference between what he gave and what he received.

RULE 2.—When the under figure is larger than the one above it, suppose 10 to be added to the upper figure, and subtract the under figure from the sum; then add 1 to the next left hand place in the subtrahend.

17. What is the difference between 115 and 2032?

Operation.

```
2 0 3 2
  1 1 5
-------
1 9 1 7
```

Explanation.—In this question, the 5 being larger than the figure above it, we say 10 added to 2 makes 12, and 5 taken from 12 leaves 7. In the second column, 1, or 1 ten, added to 1, makes 2, and 2 taken from 3 leaves 1. In the third column, the 1 being more than the cipher, we suppose 10 placed where the cipher is, and take 1 from it, which leaves 9. Although there is no figure under the fourth place in the minuend, because we added 10 into the third place, we must suppose 1 to be set under the 2, and subtracted from it.

18. If the greater of two numbers be 1661, and the less 446, what is the difference? Ans. 1215.

19. A man has in his possession $4092, but he owes $1519; how much is he worth? Ans. $2573.

20. By the census of 1820, Boston contained

43'298 inhabitants; Philadelphia, 108'161; how many more inhabitants were there in Philadelphia than in Boston? Ans. 64'863.

21. A gentleman purchased twenty-five thousand acres of wild land. After selling a certain quantity, he had eighteen thousand, eight hundred and sixty-one acres left. How many did he sell?
Ans. 6'139.

Illustration of Rule 2.—To prove the correctness of rule 2, it will be best to make use of an example.

Let it be required to subtract 478 *from* 592.

Operation 1.	Operation 2.
5 0 0+8 0+1 2=	5 9 2
4 0 0+7 0+ 8=	4 7 8
1 0 0+1 0+ 4=	1 1 4

By separating the minuend and subtrahend into parts, as in the first operation, and subtracting, we take 8 from the 2, and 1 ten which was contained in the 9 tens, or from 12; 70 from 80, and 400 from 500, and obtain the differences between all the parts of the minuend and all the parts of the subtrahend; and these differences, viz.: 4, 10 and 100, added together, must give the difference between the whole minuend and the whole subtrahend, or between 592 and 478. But as we do not generally separate the numbers for subtracting, it will be necessary to show that we obtain the same answer by the common method. In the second operation of the foregoing example, when we suppose 10 added to the 2, we may suppose this 10 to be taken from the 9 tens, and by subtracting 8 from the 12, we obtain 4, the same remainder as when we separated the numbers. In the next place, as we have subtracted from one of the tens in the 9 tens, we have now to subtract the 7 tens from the remaining 8 tens in the 9, which leaves 1 ten; but as we did not remove the 1 ten from the 9 which we supposed added to the 2, if we were to subtract the 7 from the 9, we should subtract twice from that 1 ten which we supposed removed; therefore, to balance that ten, we must add 1 ten to the 7 tens, and the difference will be the same as though we had taken 1 ten from the 9 and not added any to the 7, for the difference between 8 and 9, is the same as the difference between 7 and 8. The difference between the 4 and 5 is 1, and the whole remainder the same as found by the first operation. As the 1, which we add to the next place in the subtrahend, is just equal to the 10 which we added to the minuend, the difference must always be the difference between the given numbers, for by adding 10 to both numbers we increase both alike, there being no difference between 10 and 10.

22. Plymouth, Mass. was first settled in the year 1620; how many years from that time to the year 1826? Ans. 206.

23. Required the difference between 170’063 and 169’007. Ans. 1’056.

A short horizontal line thus — is used as a sign of subtraction, and denotes that the number after it, is to be taken from the number before it; thus, 12—8 =4, and is read, 12 minus 8 equals 4; or, 8 subtracted from 12 equals 4; the word *minus* meaning *less*.

The scholar is required to work the following examples, as directed by the sign of subtraction:

24. 5’924— 279= 5’645.
25. 75’428— 2’096=73’332.
26. 100’000— 50’009=49’991.
27. 1’961’851— 987’654=
28. 85’400’301— 98’394=
29. 506’070’809—90’706’050=

The sum of the remainders in the three last examples, is 501’640’863.

30. What is the difference between seventy thousand, three hundred and twenty-six, and ten thousand, two hundred? Ans. 60’126.

31. How much larger is two million, two thousand and two, than one million and two?

Ans. 1’002’000.

32. America was discovered in 1492; Jamestown, Va. was settled 1608, and Plymouth, Mass. in 1620. How many years from the discovery of America to the settlement of Jamestown and Plymouth?

Ans. { To the settlement of Jamestown, 116.
To the settlement of Plymouth, 128.

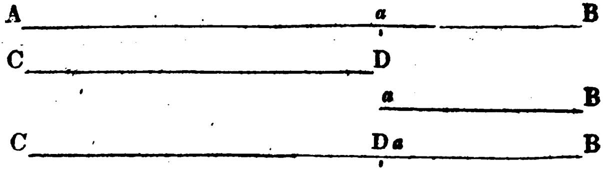
* * *

METHOD OF PROVING SUBTRACTION.

Add the remainder and subtrahend, or less number, together, the sum will equal the minuend, or greater number, if the work be right.

Illustration. The difference between two numbers added to the less, must produce a sum equal to the greater; thus, 11 taken from 15 leaves 4; 4 is what 11 wants of being as much as 15, consequently, if we add 4 to 11, the sum will be 15.

But more mathematically;—let us suppose the line A B to be longer than the line C D.



To find how much longer the line A B is than C D, we must take away a part of the line A B, which shall be as long as the line C D. Suppose the part A *a* to be as long as C D; by removing that part of A B, there will remain the part *a* B, which is the difference between the lengths of A B and C D. Now if we annex the line *a* B to C D, we should make the line C B just as long as A B.

We may suppose these lines to be any lengths, and if we wish to find the difference of their lengths, we must subtract the length of the shorter from that of the longer, and the difference would be the length of *a* B; but the length of *a* B added to the length of the shorter line, will always give the length of the longer, which would be the minuend.

33. 670135 *minus* 478103 are how many?

Operation.
670’135
478’103
192’032
Proof 670’135

Here 192032 is the difference between 670’135 and 478’103, and by adding 192’032 to 478’103, we obtain the minuend, as was proved in the Illustration.

The pupil can ascertain whether he performs the following examples correctly, by proving his work.

34. 17800 is how many less than 67987?

35. 1000404 minus 66666 equal how many?

36. 948372 minus 716253 equal how many?

37. What is the difference between 100000 and 99999, equal to?

38. What number must be added to 1’111, that the sum may be 10’000?

39. To what number must 1 be added that the sum may be one million? Ans. 999’999.

40. To what number must 100’101 be added that the sum may be one million, ten thousand and ten? Ans. 909’909.

41. A man owing 1076 dollars, paid all but 108 dollars; how many dollars did he pay? Ans. 968.

42. A clergyman being asked how long it was since he began to preach, answered that he was 69 years old, and that he began to preach at the age of 25. How many years had he been preaching? Ans. 44 years.

43. A man owing 721 dollars, paid 509; how much did he then owe? Ans. 212 dolls.

Questions to be worked by Addition and Subtraction.

1. A gentleman bought 2 pieces of land; one contained 96 acres, the other 103. If he sell 47 acres, how many will remain? Ans. 152.

2. A merchant bought a cask of molasses containing 119 gallons. He sells to one man 11 gallons; to another, 13; to another, 14. How many gallons has he left? Ans. 81.

3. One man owes another fifteen hundred dollars, to be paid as follows; one hundred and ten dollars in 3 months; two hundred and thirty-one, in 6 months; a thousand and ten, in 10 months, and the remainder at the end of one year. How much is he to pay at the end of the year. Ans. 149 dollars.

Questions to be worked by Multiplication and Subtraction.

1. What is the difference between 11 times 71 and 1,000? Ans. 219.

2. Which is the larger number, 6 times 665, or 7 times 565? Ans. 6 times 665, by 35.

3. 290×9023—6'504=2'610'166.

4. 167×7×8—3'470=5882.

5. 265—77×3=564.

6. A merchant buys 175 pounds of pork, at 7 cents a pound, and sells it for 8 cents; how much does he gain? Ans. $1,75.

7. If I purchase 1200 pounds of butter for 156 dollars, and sell it at 14 cents per pound, do I gain or lose, and how much? Ans. I gain $12.

◆

DIVISION OF SIMPLE NUMBERS.

Division is ascertaining how many times one number can be subtracted, or taken, from another.

When we take one number from another once, to find the difference between those numbers, we *subtract*; but when we take one number from another once, or more, to find how many times one number can be subtracted from another, we *divide*.

Although we subtract in Division as well as in Subtraction, the object to be obtained by dividing, is essentially different from that sought in Subtraction. In Subtraction we never subtract but once, the only object being to find how much larger one number is than the other; but in Division we may subtract once or more, the object being to find how many times one number contains another.

If the difference between 8 and 4 be demanded, we ascertain it by subtracting the 4 once from the 8; thus,

8
4
—
4 difference.

But if 8 is to be divided by 4, we subtract the 4 from the 8 as many times as it can be;

thus, 8 It appears that 4 can be taken twice
4 from 8, therefore 2 is the answer.
—
4
4
—

We may always obtain an answer to any question in Division, by subtracting the less number from the greater, and then the less number from the remainder, and so on, subtracting the less number every time from the last remainder, until the remainder becomes less than the number we are subtracting; and the number of times we subtract would be the answer. But it is easy to see that this method must be very tedious when the less number can be subtracted from the other a great number of times, refore a shorter method has been devised.

DEFINITIONS.

A Dividend is a number given to be divided.

A Divisor is a number given to divide by.

A Quotient is a number showing how many times the divisor can be subtracted from the dividend.

A Remainder is a number left after dividing; which must always be less than the divisor. [If it be required to divide 16 by 5, 16 is the dividend, 5 the divisor, and as we can subtract 5 3 times from 15, 3 is the quotient, and the 1, which is left, is the remainder.]

Let the scholar answer the following questions.

How many times can 3 be taken from 9? We found in the Multiplication Table, that 3 multiplied by 3, or 3 added together 3 times, made 9; consequently 3 can be taken from 9 as many times as there were 3's put together to make 9. How many times can 3 be taken from 12? 3 from 15? 4 from 12? 4 from 16? 5 from 10? 5 from 20? 2 from 14? 5 from 15? 3 from 15? 3 from 18? 3 from 21? 7 from 21? How many times does 24 contain 8? 24 contain 6? 32 contain 8? 36 contain 6? 42 contain 7? 42 contain 6? How many times can 9 be taken from 28, and what number will remain? 7 from 30? 7 from 46? 9 from 40? 9 from 60? 8 from 36? 8 from 47? 8 from 60? 10 from 54? 10 from 85? 9 from 100?

RULE I.

Place the divisor at the left hand of the dividend.

Ascertain as near as you can in the mind, how many times the divisor can be taken from the dividend, and place a figure expressing that number at the right hand of the dividend for the quotient.

Multiply the divisor by the quotient, and subtract the product from the dividend.

Note 1. If the remainder be as large or larger than the divisor, the quotient is too small for the divisor, in that case, could be subtracted once or more from the remainder.

If the product be larger than the dividend, the quotient is too large.

Question 1. How many times can 12 be taken from 48?

Operation by subtracting.

48 dividend.
12 divisor.
——
1st subtraction 36
12
——
2d " 24
12
——
3d " 12
12
——
4th " 00

Illustration. In this question, as we can subtract 12 4 times from the dividend, 4 is the quotient. The 36 is a part of the 48 left after subtracting the 12 once; the 24 is what remains of the 36 after taking out 12; consequently it is a part of the 48; and the 12, from which we subtract the last time, is that part of 24 remaining, after taking 12 from 24; therefore in each of the subtractions, we subtract from 48, or some part of it. Now were we to add the 12 together as many times as we have subtracted it, the sum would be 48. But were we to multiply 12, the divisor, by 4, the number of times we have subtracted, the product would be the same number that we found by adding the 12, 4 times together; and if we subtract the 48, made by multiplying the 12 by 4, we should subtract 12 as many times as we did when we subtracted only 12 at a time, as may be seen by the following operation.

Operation by Division.

Divisor 12) 48 (4 quotient.
48
——
0

By comparing this operation with the preceding one, we see how much easier it is to perform questions by dividing than it would be were we obliged to perform the same operations by subtracting.

2. Divide 99 by 11. Quotient 9.
3. Divide 128 by 16. Quot. 8.
4. How many times can 13 be taken from 65? Ans. 5.

6. What is the quotient of 176, divided by 18?

Operation.

18)176(9
  162
  ---
   14 remainder.

7. Divide 115 by 19. Quotient 6, Remainder 1.
8. " 127 " 25. " 5 " 2.
9. " 257 " 127. " 2 " 3.
10. " 693 " 231. " 3 " 0.
11. " 928 " 231. " 4 " 4.
12. " 7795 " 1111. " 7 " 18.

13. A boy wishes to divide 35 pears equally between 5 companions: how many must he give each?

Operation.

5)35(7 Ans.
  35

Explanation. Were the boy to give 1 pear to each companion, and then 1 more to each, and so on, until he had distributed the whole number, he would evidently give each companion an equal number. By taking 5 pears from the whole number, we take away enough to give one to each boy: by taking away 5 more, we take enough to give each boy another, then each boy will have 2; consequently, as many times as we can take 5 from 35, the whole number of pears, so many each boy will receive; and as 7 is the number of times we can subtract 5 from 35, 7 is the number which each boy is to receive.

Note 2. It appears from what has been shown in the explanation of the last question, that when any number or quantity, is to be divided into equal parts, or equally between several persons, that the number of parts into which such a number or quantity is to be divided, must be the divisor, and that the quotient will show what one of those parts is.

14. If 66 be divided into 22 equal parts, what will be one of those parts? Ans. 3.

15. Divide 696 dollars equally between 116 men; how many dollars will each man receive? Ans. 6.

16. What is the 25th part of 225? Ans. 9.

17. What is the 9th part of 81? Ans. 9.

18. How many bushels of wheat, at 9 shillings a bushel, can be bought for 54 shillings? Ans. 6.

19. An army consisting of 9702 men, receives 67914 pounds of bread per week; how many pounds does each man receive? Ans. 7.

RULE. II.

When it is found that the quotient is more than 9, ascertain how many times the divisor can be taken from an equal number of figures at the left hand of the dividend, or one more, if an equal number be not so large as the divisor; set a figure representing the number of times for the first figure of the quotient; multiply the divisor by this figure, and subtract the product from the figures taken at the left hand of the dividend.

To the right hand of the remainder bring down the next figure of the dividend; divide this number as directed by the first rule; and continue the same process till all the remaining figures of the dividend are brought down.

If the number made by the remainder and figure brought down, be not so large as the divisor, place a cipher in the quotient and bring down the next figure.

20. Required the quotient of 2341, divided by 15.

Operation.

```
15)2341(156
   15
   ---
    84
    75
    ---
     91
     90
     ---
      1
```

Explanation. In this question, we find that the divisor can be taken only once from the two first figures of the dividend, therefore we place 1 for the first figure of the quotient, multiply the divisor by it, and subtract the product from the 23. To 8, the remainder, we bring down 4, the next figure in the dividend, and divide the 84 by 15, which gives 5 the next figure in the quotient. We then bring down 1, the remaining figure of the dividend to the right hand of 9 the remainder, and find that 15 can be taken 6 times from the 91. We then multiply the divisor by 6, and subtract the product from 91, which leaves 1 for a remainder.

21. Divide 6556 by 32.

Operation.

32)6556(204
64
———
156
128
———
28

Explanation. After bringing down the 5 to the right of the 1, 15, the number formed, is not so large as the divisor, consequently, we cannot subtract the divisor from it; we therefore, place a cipher in the quotient, and to the right of the 5 bring down 6, the next figure of the dividend.

22. Divide eight million, four hundred ninety thousand, nine hundred and sixty-five, by thirty-four thousand, six hundred and fifty-seven.

Quotient 245.

23. How many times can nine thousand, eight hundred and seventy-six, be subtracted from two hundred twenty-seven thousand, one hundred and forty-eight? Ans. 23.

24. A gentleman has a field of corn containing nine thousand one hundred and two hills; each row has two hundred and twenty-two hills; how many rows in the field? Ans. 41.

25. What is the quotient of 484848, divided by 1036? Ans. 468.

26. A bookseller purchased 591 arithmeticks for $437,34, what does he give a piece?

Ans. 74 cents.

Note 3. Dollars and cents, when both are given, are divided in the same way as simple numbers, when the divisor is a simple number; but the two right hand figures of the quotient, must be separated, which will be cents.

27. How many arithmeticks at 74 cents a piece can be bought for $437,34? Ans. 591.

28. If a person spend $101,40 in one year, what does he spend per week, there being 52 weeks in a year? Ans. $1,95.

Illustration of Rule 2. The quotient will always contain one more place of figures than there are in the dividend to the right of the product of the first quotient figure. Although we multiply the divisor by each figure in the quotient, as though it were in units' place, by placing the product as far towards the left of the dividend as the quotient figure is from units' place in the quotient, we really subtract the divisor as many times from the dividend as there are units in the quotient figure.

Let it be required to divide 5775 *by* 25.

Operation.

```
2 5)5 7 7 5(2 3 1
    5 0
    ———
    7 7
    7 5
    ———
      2 5
      2 5
      ———
```

There will be 3 figures in the quotient the 2 therefore is really 200, and by placing the product of 25 multiplied by 2 under the 57, or really 5700, it becomes 5000, which is the product of 25 multiplied by 200, consequently we subtract 25 the divisor, 200 times from a part of the dividend. As 75 belong to the right hand of 7 the remainder, there remains 775 yet to be divided. The product of 25 multiplied by 3, is 75, but as the 3 in the quotient is really 3 tens, the product must be increased by 10, and this we do, by placing 5, the right hand figure under 7 which was brought from tens' place in the dividend; therefore we now subtract 25, 30 times from another part of the dividend. We have now subtracted the whole of the dividend except 25, and as that is just equal to the divisor, the divisor can be taken from it but once; therefore when division is correctly performed, we place an unit in the quotient every time we subtract the divisor from the dividend.

29. Bought 7860 yards of cloth for $5266,20; what did it cost per yard? Ans. 67 cents.

30. If in the preceding example the whole cost of the cloth had been $6602,40 what would have been the cost per yard? Ans. 84 cents.

31. A general has an army of 97'440 men, which he wishes to divide into 14 divisions; how many men in each division? Ans. 6'960.

PROOF.

Multiply the divisor by the quotient and add the remainder, if any, to the product; the product or the sum, will equal the dividend, if the work be right.

Illustration. By dividing, we subtract the divisor from the dividend as many times as we put an unit in the quotient, consequently were we to add the divisor together as many times as we subtracted it from the dividend, the sum must equal the dividend, if there be no remainder; but as multiplying the divisor by the quotient is the same as adding it together as many times as the quotient contains an unit, the product must equal the dividend. When a remainder occurs, it is what is left over the number which we subtracted from the dividend, and by adding it to the product of the divisor into the quotient, the sum must equal the dividend.

A short horizontal line between two points is used as a sign of division, and is frequently expressed by the word *by*. It shows that the number before it is to be divided by the number after it.

Thus $12 \div 6 = 2$; and is read 12 divided by 6 is equal to 2; or 12 by 6 equals 2.

Let the scholar perform the following questions as directed by the sign and prove them.

32. 8'364÷12. 35. 114'708 by 3'476.
33. 3'569'025÷10'345. 36. 270'404 by 2'601.
34. 639'819'108÷2'036. 37. 1'236'270 by 609.

Remark. When a remainder occurs in division, it is usually written at the right hand of the quotient with the divisor below it, separated by a line drawn between them. The remainder and divisor thus written, is called a fraction.

38. How many times can 12 be subtracted from 625?

Operation.
1 2) 6 2 5 (5 2
6 0
——
2 5
2 4
——
1

Explanation. 12 can be subtracted from 625, 52 times and 1 remains. As there is a remainder, the 52 is not the exact quotient, we therefore write the remainder 1 at the right hand of the 52 and the divisor below it. We read this quotient, thus, *fifty-two and one twelfth.*

39. What number must be multiplied by 47 to make a product of 4'089'423? Ans. 87'009.

40. A gentleman said he owned an equal part with several others of a factory valued at $244'125, and that his share was worth $6'975; how many owners to the factory? Ans. 35.

41. A factory worth $376'569 is owned equally by 47 men; what is the share of each?
Ans. $\$8'012\frac{5}{47}$.

42. A merchant has 7'800 bushels of salt, which he wishes to put into bins of 121 bushels each; how many will be required? Ans. $64\frac{56}{121}$.

* * *

CONTRACTIONS.

When the divisor contains but one figure, or when a *cipher* or ciphers stand at the right hand of the divisor, the operations in division may be contracted or shortened.

Contraction 1.

When the divisor consists of but one figure, the multiplying and subtracting are performed in the mind, and the quotient set directly under the dividend. This method is usually called *Short Division.*

43. What is the quotient of 940, divided by 4?

Operation 1.

4) 9 4 0

2 3 5

Operation 2.

4) 9 4 0 (2 3 5
8
1 4
1 2
2 0
2 0

Illustration of contraction 1. Twice 4 are 8, we can take 4 from 9, 2 times, as in the 1st operation, and 1 remains. We next suppose the 1 written at the left hand of the 4, which makes 14; then 4 can be taken from 14, 3 times, and 2 remains. Lastly, suppose the 0 written at the right of the remaining 2, which makes 20; then 4 can be taken from 20, 5 times, and nothing remains. The remainders are called so many tens, because they belong to the left of the next figure. By comparing the first with the second operation, it will be seen that the numbers which we divide in the mind in the first, are the same as those formed by bringing down the next figures of the dividend in the second operation.

44. Divide 9'600'120 by 4. Quot. 2'400'030.
45. What is a third part of 76'596?
Ans. 25'532.

46. Divide 165'217 by 3.

Operation.

$$\begin{array}{r} 3\,)\,\underline{165'217} \\ 55'072\tfrac{1}{3} \end{array}$$ Quotient.

47. How many yards of cloth, at $5 a yard, can be bought for $95? Ans. 19.

Note.—There will be as many yards as there are 5's in 95.

48. What is a 6th part of 65'057? Ans. 10'842$\frac{5}{6}$.

49. What is the eighth part of one thousand? Ans. 125.

50. A gentleman divides his estate, amounting to 11'097 dollars, equally between nine children: what does each receive? Ans. $1'233.

51. How many spaces of time, each 5 minutes, in 120 minutes? Ans. 24.

52. If a stick of timber 89 feet long, be divided into 7 equal parts, what will be the length of each part? Ans. 12$\frac{5}{7}$ feet.

53. If a stick of timber 100 feet long, be divided into divisions of 8 feet each, how many divisions will it consist of? Ans. 12$\frac{4}{8}$.

Contraction 2.

When any of the places at the right of the divisor, are occupied by ciphers, they may be omitted, and an equal number of figures must be omitted at the right of the dividend. The figures cut off from the dividend, must be brought to the right of the remainder, if any, if not, they will constitute the whole remainder.

54. What is the quotient of 8'990'123, divided by 76'300?

Operation.

```
76300)8990123(117
      76300
      ------
      136012
       76300
      -------
       597123
       534100
       ------
        63023
```

Operation by the contraction.

```
763|00)89901|23(
   763)89901(117
       763
       ----
       1360
        763
       ----
        5971
        5341
        -----
         63023
```

Illustration of contraction 2.—We obtain the same quotient, whether we omit the ciphers and an equal number of figures at the right hand of the dividend, or retain them, because the ciphers on the right of the divisor, will always come under an equal number of figures brought from the right of the dividend; and as they do not lessen the figures above them by subtracting, the figures above them the last time we subtract, which were brought from the right hand of the dividend, will always be left as a remainder, or a part of it.

By examining the two last operations, it will be seen that we obtain the same quotient and the same remainder, by both methods.

The scholar is required to perform the following examples, as directed by the sign of division.

55. 4’521’084÷3’600=1’255, and 3’084 rem.

56. 9’876’543’210÷432’000=22’862, and 159’210 rem.

57. 2’468’013’579÷999’000=2’470, and 483’579 rem.

58. 5’489’074’426÷500’000=10’978, and 74’426 rem.

59. 8’296’547’000÷9’000’000=921, and 7’547’000 rem.

60. 98’899’887’701÷9’001’000=10’987, and 5’900’701 rem.

Contraction 3.

When the divisor is 1 with a cipher, or ciphers, to the right hand, we have only to cut off as many figures at the right hand of the dividend, as there are ciphers in the divisor;—the figures cut off will be the remainder, and the others, the quotient.

61. Divide 123’456’789 by 10’000.

Operation by the common method.

```
10000)123456789(12345
      10000
      -----
       23456
       20000
       -----
        34567
        30000
        -----
         45678
         40000
         -----
          56789
          50000
          -----
           6789 Rem.
```

Contraction.

Divisor, Dividend,
1 0 0 0 0) 1 2 3 4 5 6 7 8 9

Quotient, Rem.
1 2 3 4 5 6 7 8 9

Illustration of contraction 3.—The figure 1 in the divisor, can be taken from every figure in the dividend as many times as there are 1's in the figures of the dividend; consequently, the figures in the quotient must be the same as the left hand figures of the dividend. As the ciphers in the divisor will come under an equal number of figures at the right hand of the dividend, those figures will always remain. See the operation and contraction of the last example.

62. What is the quotient of ninety thousand and nine, divided by one thousand. Ans. $90\frac{9}{1000}$.

63. Divide one million and one, by one million. Quot. $1\frac{1}{1000000}$.

64. A gentleman said that he owned a tenth part of a tract of land, valued at $4'603; what is the value of his share? Ans. $$460\frac{3}{10}$.

65. What is a 100th, 1000th, 10[illegible]000th part of 745'638'965?

Ans. $\begin{cases} 7'456'389\frac{65}{100}. \\ 745'638\frac{965}{1000}. \\ 7'456\frac{38965}{100000}. \end{cases}$

Questions to be worked by Addition and Division.

1. A farmer sells 21 barrels of cider for $41, and 19 barrels for $39; what does the cider average a barrel? Ans. $2.

2. A marketman sells 3 turkeys, weighing 9, 10, and 13 pounds, for $3,52; what does he receive per pound? Ans. 11 cents.

3. From the creation of the world to the birth of Christ, was 1'461'460 days; from the birth of Christ to January, 1826, was 666'490 days. Estimating

365 days in a year, how many years from the creation to January, 1826? Ans. 5'830.

Questions to be worked by Multiplication and Division.

1. A gentleman divided 4 shares of canal stock equally between 5 sons. Now, if each share is worth \$597, what does each son receive? Ans. \$477$\frac{3}{5}$.

2. A man buys 165 pounds of butter for 15 cents a pound. In market he sells it for 17 cents a pound, and receives for the whole \$27,71; by which he finds it had lost in weight: how many pounds did he sell? and what did he give for the whole?

Ans. 163 pounds; he gave \$24,75.

3. A merchant sells 3 chests of tea, each weighing 96 pounds, for \$118,08; what does he receive a pound? Ans. 41 cents.

Questions to be worked by Subtraction and Division.

1. If the difference between 307'199 and 126'900, be divided into 9 equal parts, what number will one of the parts be? Ans. 20033$\frac{2}{9}$.

2. Massachusetts contains 7'250 square miles, and 529'250 inhabitants; New-York, 46'000 square miles, and 1'610'000 inhabitants. Which State contains the greatest number of inhabitants to a square mile?

Ans. Mass. contains 38 more than N. Y.

Questions to be worked as directed by the signs.

1. $10732 + 720 - 67 \times 13 \div 110 = 1345\frac{55}{110}$.

2. $672 \div 7 \div 6 + 2 - 6 \times 4 \div 6 = 8$.

3. A boy told his companion, that he had apples enough to equal $18 \times 2 \div 3 \div 12 \times 3 + 2 \div 5 + 1$; how many had he? Ans. 2.

4. A gentleman being asked how many dollars his daily income was, answered, that a 10th part of 40 plus 2 into 4 by 2 minus 8 by 4, would be the number; what was his daily income? Ans. \$1.

SECTION IV.

COMPOUND NUMBERS.

We have thus far treated of the most simple methods of collecting and separating Simple Numbers; we shall next explain the corresponding methods of collecting and separating Compound Numbers.

A Compound Number is one in which an unit in one column does not represent the same part of the same quantity, as another unit standing in some other column of the same number.

EXAMPLE.

days.	hours.	minutes.	seconds.
1	1	1	1

In this example every figure denotes time, but no two units in different columns, express the same space of time. The 1 at the right hand is 1 second, and the 1 in the next column represents 60 seconds; but were it a simple number, the 1 in the second place, would stand for 10 seconds. If all the figures in the above example, denoted seconds, minutes, or hours, &c. the number would be a simple one. 1111 seconds is a simple number, because the 1 standing in the second place, equals 10, and the 1 in the third place, equals 100 seconds.

DEFINITIONS.

Abbreviation is one letter or more standing for a whole word; as y. for years, gal. for gallons, &c.

Denomination. By a denomination is meant those figures which represent that part of a quantity ascertained by any particular weight or measure.

EXAMPLE.

tons.	hundreds.	quarters.	pounds.	ounces.	drams.
2	12	3	21	15	11

The words, *tons*, *hundreds*, *quarters*, *pounds*, *ounces*, *drams*, are the names of the denominations in one manner of ascertaining the weight of several kinds of goods, and the figures below them express so many units of each particular denomination. Each denomination is a simple number by itself, because the units expressed by one figure, are equal to the same number of units contained in any other figure of the same denomination. In the denomination of drams, the right hand figure denotes one dram, and the 1 in the 2d place, denotes 10 drams. But the whole number taken together, that is, the tons, hundreds, quarters, &c. is a compound one, for an unit in the denomination of drams, is not equal to an unit in either of the other denominations, as an unit in either of the other denominations, expresses a greater quantity.

Remark.—From what has been said, we infer that we cannot add and subtract compound numbers, in every respect by the same methods that we did simple numbers; yet the general principles in both cases are the same. As 10 in one denomination, does not equal 1 in a higher denomination, when we obtain 10 by adding up any denomination, we cannot add one to the next denomination, as in simple numbers; but to obtain the exact amount, we must add 1 to a higher denomination for as many of the less, as it takes to equal 1 in the higher. In simple numbers we carry for 10; in compound numbers we carry for some other number, and this is all the essential difference between the operations in simple and compound numbers.

ADDITION OF COMPOUND NUMBERS.

Addition of compound numbers is collecting two or more compound numbers, so that the several numbers may be expressed by one.

RULE.

State the question by setting down one of the numbers, and all the others below it in such a manner, that every denomination may stand under the place of its own name in the upper number, having the least denomination at the right hand.

Add the least denomination the same as in simple numbers, and carry one to the place of the next higher denomination for as many as it takes of the denomination added, to equal one in the next higher, and set down what remains. Proceed in the same manner with all the other denominations.

Note 1. If the greatest denomination in the given numbers, be the highest denomination in that particular weight or measure, we add them in all respects as simple numbers.

TABLE OF ENGLISH MONEY.

		Abbreviation.			Abbreviations.
4	Farthings,	*qr.*	make	1 Penny,	*d.*
12	Pence,		"	1 Shilling,	*s.*
20	Shillings,		"	1 Pound,	£

Question 1. What is the sum of 135£. 18 s. 9 d. 2 qr., 25£. 12 s. 1 qr., and 100£. 15 s. 1 d. 3 qr.?

Operation.

	£	*s.*	*d.*	*qr.*
	135	18	9	2
	25	12	0	1
	100	15	1	3
Ans.	262	5	11	2

Illustration.—By adding up every denomination of the given numbers, we must obtain a compound number, which shall equal all the given numbers. In the last question, the 2 farthings, which we set in the place of farthings, and the 1 penny which we carried to the column of pence, (which is equal to the other 4 farthings,) equal the number of farthings in the three given numbers; the 11 pence are equal to all the pence in the three given numbers, together with the 1 penny which was brought from the farthings; the 5 shillings are equal to the three given numbers of shillings, except the 40, which were carried to the pounds; and the 262 pounds equal the three given numbers of pounds, together with the two which was brought from the shillings; therefore 262 £. 5 s. 11 d. 2 qr. equal all the denominations of the three given numbers.

2. Add together 11 £. 19 s. 3 qr., 2600 £. 11 d. 2 qr., 111 £. 17 s. 11 d. 1 qr.

Operation.

£.	*s.*	*d.*	*qr.*
11	19	0	3
2600	00	11	2
111	17	11	1
2723	17	11	2

Note 2. The best way, in stating compound numbers for adding or subtracting, is to set down the abbreviations in the same order as they stand in the tables, and it will then be easy to discover in what place the given denominations must be set. It often happens with beginners, that when there is no number given in any particular denomination, they place the number in the next less denomination in that place which must produce an error. Where there is no number given in any denomination, the place of that denomination should be supplied with ciphers. In the last example, had we placed the 3 qr. where the cipher is, it would have read 3 *pence* instead of 3 *farthings*, and of course a mistake would have occurred.

3. Suppose a gentleman owes to one creditor 160 £. 2 s. 7 d. 1 qr.; to another, 29 £. 13 s. 6 d. 3 qr; to another, 110 £. 1 s. 1 d. 1 qr.; what is the sum of the three debts? Ans. 299 £. 17 s. 3 d. 1 qr.

TROY WEIGHT.

		Abbreviation.				Abbreviations.
24	Grains	*gr.*	make	1	Pennyweight,	*pwt.*
20	Pennyweights		„	1	Ounce,	*oz.*
12	Ounces		„	1	Pound,	*lb.*

By this weight are weighed gold, silver, jewels, electuaries, and all kinds of liquor.

4: What is the sum of 21 lb. 11 oz. 13 pwt. 4 gr., 207 lb. 9 pwt. 22 gr., and 10 lb. 9 oz. 19 gr.?

lb.	*oz.*	*pwt.*	*gr.*	
21	11	13	4	
207	00	9	22	
10	9	00	19	
239	9	3	21	Ans.

5. A goldsmith sells a silver tea-pot, weighing 1 lb. 2 oz. 7 pwt. 11 gr.; a sugar bowl, weighing 11 oz. 15 pwt. 9 gr.; a creamer weighing 9 oz. 23 gr.; and 6 spoons weighing 1 lb. 1 oz. 1 pwt. 1 gr.; what is the weight of the whole?

Ans. 4 lb. 0 oz. 4 pwt. 20 gr.

APOTHECARIES WEIGHT.

20	Grains,	*gr.*	make	1 Scruple, *sc.*
3	Scruples		,,	1 Dram, *dr.*
8	Drams		,,	1 Ounce, *oz.*
12	ounces		,,	1 Pound, *lb.*

Apothecaries and physicians mix their medicines by this weight; but drugs are bought and sold by avoirdupois.

(6.)

lb.	*oz.*	*dr.*	*sc.*	*gr.*
6	9	7	1	15
2	11	5	2	16
13	6	1	1	3
23	3	6	2	14

7. What is the sum of 7 lb. 3 dr.; and 14 lb. 19 gr?

Ans. 21 lb. 0 oz. 3 dr. 0 sc. 19 gr.

AVOIRDUPOIS WEIGHT.

16	Drams,	*dr.*	make	1 Ounce, *oz.*
16	Ounces		,,	1 Pound, *lb.*
28	Pounds		,,	1 Quarter, *qr.*
4	Quarters		,,	1 Hundred, *cwt.*
20	Hundreds		,,	1 Ton. *T.*

All coarse and drossy goods, groceries, tallow, bread, hay, leather, and all metals, except gold and silver, are weighed by this weight.

(8.)

T.	cwt.	qr.	lb.	oz.	dr.
89	16	1	27	15	11
200	12	2	10	11	15
	1	2	3	4	5
290	10	2	13	15	15

Note 3. As it is difficult to ascertain in the mind how many times the number which we carry for, can be taken from a large number, the most expeditious method is to divide the sum of the numbers in those denominations where it is large, by as many of that denomination as are equal to 1 in the next higher, and the quotient will be the number to be added in the next place.

9. What is the weight of 3 hogsheads of sugar, the first weighing 10 cwt. 20 lb.; the 2d., 9 cwt. 1 qr. 15 oz.; and the 3d., 11 cwt. 15 lb. 14 dr.?

Ans. 1 T. 10 cwt. 2 qr. 7 lb. 15 oz. 14 dr.

10. A merchant imports in three ships the following quantities of iron; one hundred and twenty tons, six hundred; fifteen tons, three quarters, twenty-two pounds; and twelve tons, thirteen hundred, 1 quarter, twenty-five pounds, ten ounces and nine drams. What is the weight of the whole?

Ans. 148 T. 1 qr. 19 lb. 10 oz. 9 dr.

Remark.—The practice of estimating 28 pounds as a quarter of a hundred, was introduced into business that the buyer of coarse articles should have 100 pounds of a quantity free from waste; but as many people have now adopted the more consistent method of making their calculations by the true hundred, we shall insert a table for that purpose.

NEAT WEIGHT.

16	Drams,	*dr.*	make	1	Ounce,	*oz.*
16	Ounces		"	1	Pound,	*lb.*
25	Pounds		"	1	Quarter,	*qr.*
4	Quarters		"	1	Hundred,	*cwt.*
20.	Hundreds		"	1	Ton,	*T.*

11. A farmer sells 3 loads of hay, weighing 19 cwt. 2 qr. 17 lb.; 16 cwt. 16 lb. 15 oz.; 17 cwt. 3 qr. 1 oz.; what is the weight of the whole?

Ans. 2 T. 1 cwt. 23 qr. 9 lb.

13. A teamster's load consists of the following articles, viz. 1 hogshead of molasses, weighing 9 cwt. 20 lb.; a cask of salt, 7 cwt. 1 qr. 14 oz.; 2 bales of cotton, 4 cwt. 17 lb.; a pipe of brandy, 6 cwt. 3 qr. 11 oz. 4 dr.; a box of dry goods, 1 cwt. 13 oz. 12 dr.; and a cask of coperas, 3 qr. 24 lb.; What is the weight of his load?

Ans. 1 T. 9 cwt. 1 qr. 13 lb. 7 oz.

Note 4.—All questions in which the word *neat* is used, will be operated by the last table.

PROOF.

After adding up the several denominations in the usual way, add them from the top downwards; if the work has been performed correctly, the sums of the several denominations will be the same as before, for we only add the same numbers in a different way.

LONG MEASURE.

12 Inches,	*in.*	make	1 Foot,	*ft.*
3 Feet		"	1 Yard,	*yd.*
5½ Yards, or 16½ Feet		"	1 Rod, Perch or Pole,	*r.p.*
40 Rods		"	1 Furlong,	*fur.*
8 Furlongs		"	1 Mile,	*m.*
3. Miles		"	1 Leagues,	*leg.*

This measure respects length only.

(13.)

leg.	*m.*	*fur.*	*r.*	*yd.*	*ft.*	*in.*
17	2	7	30	4	2	2
20	1	5	27	5	1	8
25	2	4	6	4	2	9
64	1	1	25	4	0	7

Explanation.—In this example, we find 15 in the column of yards, and we wish to know how many times we can take 5½ from it. To effect this, we multiply the 5 by the under figure in the fraction, and add the upper figure to the product. We then multiply the 15 by the same number, and divide one product by the other. The remainder must be divided by the same number by which we multiply. The same method may be pursued when any other fraction occurs; that is, multiplying both numbers by the under figure of the fraction, and dividing the remainder by it. It must be remembered, that the upper figure of the fraction is not to be added to the product, when we multiply the dividend.

```
 5½ ) 1 5
 2      2
 ---   ---
1 1 ) 3 0 ( 2
      2 2
      ---
        8
```

14. A certain field is enclosed by 3 fences of the following lengths, viz.: the first, 2 fur. 11 r. 15 ft. 2 in.; the second, 1 m. 1 fur. 38 r. 14 ft.; and the third, 6 fur. 21 r. 12 ft. 5 in. What is the distance round the field? Ans. 2 m. 2 fur. 32 r. 8 ft. 7 in.

15. A ship sailed east, 2 leg. 2 m. 7 fur. 13 r. 4 yd. 1 ft. 11 in.; S. E. 4 leg. 2 m. 33 r.; south, 9 leg. 1 m. 2 fur. What was the distance sailed?

Ans. 17 leg. 2 fur. 6 r. 4 yd. 1 ft. 11 in.

16. Estimating the distance from Boston to the city of N. Y., at 244 m. 5 fur. 36 r.; from the city of N. Y. to the city of Washington, at 240 m. 39 r.; and from Washington city to Charleston, S. C. at 550 m. 3 fur.; what is the distance from Boston to Charleston, S. C.? Ans. 1035 m. 1 fur. 35 r.

SUPERFICIAL, LAND, OR SQUARE MEASURE.

144	Square Inches, *in.*	make	1 Square Foot,	*ft.*
9	Square Feet	"	1 Square Yard,	*yd.*
30¼	Square Yards, or 272¼ Square Feet	"	1 Square Rod,	*r.*

40	Square Rods	make 1	Rood,	*R.*
4	Roods, or Square Rods	" 1	Acre,	*a.*
160				
640	Acres	" 1	Square Mile,	*m.*

This measure is used for measuring land and all other things, which have length and breadth, without any regard to thickness.

Note 5.—A square inch, foot, or yard, &c. is a square surface which has 4 equal sides, each side being 1 inch, 1 foot, or 1 yard in length. The number of square inches contained in a square foot, or the number of square feet in a square yard, is found by multiplying the number of inches, feet, &c. contained in 1 foot, or 1 yard in *Long Measure*, into themselves.

(17.)

r.	*ft.*	*in.*
35	169	101
25	202	29
17	37	127

(18.)

a.	*R.*	*r.*	*yd.*	*ft.*	*in.*
3	1	26	24	6	128
2	3	00	00	7	130
	2	11	30	1	11

19. A gentleman's farm is divided into wood land, 20 a. 2 R. 6 r. 162 ft. 100 in.; tillage, 19 a. 1 R. 7 r. 19 ft. 98 in.; pasturage, 56 a. 3 R. 4 r. 200 ft. 77 in.; mowing, 21 a.; and garden, 2 R. How much land in the farm?

Ans. 118 a. 18 r. $109\frac{3}{4}$ ft. 131 in.

20. The floors of 4 rooms in a certain house, cover 5 r. 24 in. of land; the remaining room, 1 r. 1 yd. 1 ft.; and the walls and chimney cover 2 r. and 11 in. How much land does the whole house occupy?

Ans. 8 r. 1 yd. 1 ft. 35 in.

SOLID, OR CUBICK MEASURE.

1728	Solid or Cubick inches	make 1 Solid or Cubick Foot,	*ft.*
27	Solid Feet	make 1 Solid Yard,	*yd.*
40	Solid Feet of round timber, or	make 1 Ton,	*T.*
*50	Solid Feet of hewn timber,		
128	Solid Feet	make 1 Cord of wood,	*C.*

---

* A ton of hewn timber contains 10 ft. more than a ton of round timber, because 10 ft. in the round timber is allowed for the weight *of the segments, or slabs.*

This measure is used for measuring all things which have length, breadth, or thickness.

The number of solid inches contained in a solid foot, is found by multiplying the number of inches in a square foot by 12, the number of inches in a foot. The number of cubick feet contained in a cubick yard, is found by a similar method.

21. In 3 piles of wood containing 21 C. 109 ft. 1004 in.; 17 C. 1719 in.; and 49 C. 119 ft. 13 in.; how much wood?

C.	*ft.*	*in.*
21	109	1004
17	000	1719
49	119	13

22. A merchant sells 4 lots of hewn timber, which measure 97 T. 24 ft.—103 T. 47 ft. 1001 in.,—17 T. 24 in.,—and 58 T. 1 ft. 1000 in.; how much timber in the 4 lots?

Ans. 276 T. 23 ft. 297 in.

23. How much round timber in five lots; No. 1 measuring 1001 T.; No. 2, 812 T. 293 in.; No. 3, 407 T. 37 ft. 1723 in.; No. 4, 333 T. 30 ft. 7 in.; and No. 5, 17 T. 17 ft. 17 in.

Ans. 2572 T. 5 ft. 812 in.

TIME.

60 Seconds,	*s.*	make	1 Minute,		*m.*
60 Minutes,		"	1 Hour,		*h.*
24 Hours,		"	1 Day,		*d.*
7 Days,		"	1 Week,		*w.*
4 Weeks,		"	1 Month,		*mo.*
13 Mo's, 1d.& 6 h.or 365¼ Days		"	1 Year,		*y.*

The year is also divided into 12 Calendar months, viz.;

January,	1st month, has	31 days,		July,	7th month, has	31 days,
February,	2d "	28 "		August,	8th "	31 "
March,	3d "	31 "		September,	9th "	30 "
April,	4th "	30 "		October,	10th "	31 "
May,	5th "	31 "		November,	11th "	30 "
June,	6th "	30 "		December,	12th "	31 "

When the year can be divided by 4 without a remainder, it is called leap year, in which February has 29 days.

Note 6.—Although 13 months is not an exact year, it will be sufficiently accurate in the following questions to carry for that number.

24. Add together 25 y. 9 mo. 2 w. 6 d. 41 m. 41 s. and 11 y. 7 mo. 1 w. 3 d. 20 h. 39 m. 39 s.

y.	*mo.*	*w.*	*d.*	*h.*	*m.*	*s.*
25	9	2	6	00	41	41
11	7	1	3	20	39	39

25. Estimating the time from the discovery of America to the settlement of Massachusetts, at 128 y. 1 mo. 3 w. 6 d. 23 h. (which is very near the truth;) from the settlement of Massachusetts, to the declaration of independence, at 155 y. 6 mo. 1 w. 6 d. 48 s.; how long from the discovery of America to the declaration of independence?

Ans. 283 y. 8 mo. 1 w. 5 d. 23 h. 48 s.

26. A certain note was on interest 2 y. 48 m. when a part of the note was paid. After the payment it was 2 y. 3 mo. 3 w. 5 d. 20 h. 45 s. before the note was taken up. How long from the time the note was given until it was taken up?

Ans. 4 y. 3 mo. 3 w. 5 d. 20 h. 48 m. 45 s.

CIRCULAR MEASURE OR MOTION.

60	Seconds,	"	make	1	Minute,	'
60	Minutes,	'	"	1	Degree,	°
30	Degrees,		"	1	Sign,	*s.*
12	Signs, or 360 Degrees,		"	A	Circle,	*C.*

This measure is used in reckoning latitude and longitude; in computing the revolutions of the planets; and in measuring most other circles.

Remark.—Every circle, whether large or small, is supposed to be divided into 360 degrees, consequently, a degree in a large circle, must be longer than one in a small circle; and the minutes and seconds must vary accordingly.

(27.)

°	'	"
179	51	29
28	47	53
25	25	25

Note 7.—Degrees, in most cases, are considered the highest denomination.

28. The latitude of Boston is 42° 23' 2" north, and that of Rio Janeiro, 22° 58' 59" south. How many degrees of latitude between the two places?

Ans. 65° 22' 1".

29. The longitude of Philadelphia is 75° 9' west, and that of Rome, 12° 29' east, and that of Moscow 37° 38' east; how many degrees of longitude between Philadelphia and Rome,—also between Philadelphia and Moscow?

Ans. { Between Phil. and Rome, 87° 38'.
Between Phil. and Moscow, 112° 47'.

CLOTH MEASURE.

2¼	Inches,	*in.*	make	1	Nail,	*na.*
4	Nails,		"	1	Quarter of a Yard,	*qr.*
4	Quarters,		"	1	Yard,	*y.*
3	Quarters,		"	1	Ell Flemish,	*E. Fl.*
5	Quarters,		"	1	Ell English,	*E. E.*
6	Quarters,		"	1	Ell French,	*E. Fr.*

Ells are measures used in several countries of Europe for measuring cloth; but in the United States, the *yard* is used.

(30.)

y.	*qr.*	*na.*
25	3	3
21	1	2
102	0	0

31. Bought at Amsterdam, 3 pieces of camlet; No. 1, measuring 27 E. Fl. 2 qr. 3 na.; No. 2, 25 E. Fl. 1 qr. 2 na.; No. 3, 21 E. Fl. 2 qr. 3 na. How much cloth in the three pieces?

Ans. 75 E. Fl. 1 qr.

32. A merchant buys in London 4 bales of broadcloth; the 1st contains 106 E. E.; the 2d, 102 E. E. 3 na.; the 3d, 110 E. E. 4 qr.; and the 4th, 92 E. E. 3 na. How many Ells in the 4 bales?

Ans. 411 E. E. 2 na.

33. Bought 3 pieces of silk at Paris, measuring 31 E. Fr. 1 na.; 29 E. Fr.; 28 E. Fr. 5 qr. 3 na.; how much silk in the 3 pieces?

Ans. 89 E. Fr.

WINE MEASURE.

4	Gills,	*gl.*	make	1	Pint,	*pt.*
2	Pints,		"	1	Quart,	*qt.*
4	Quarts,		"	1	Gallon,	*gal.*
31½	Gallons,		"	1	Barrel,	*bar.*
63	Gallons,		"	1	Hogshead,	*hhd.*
2	Hogsheads,		"	1	Pipe,	*P.*
2	Pipes, or 4 Hogsheads,			1	Tun,	*T.*

Spirits, perry, cider, mead, vinegar, honey and oil are measured by this measure.

(34.)

T.	*P.*	*hhd.*	*gal.*	*qt.*	*pt.*	*gl.*
11	1	1	50	3	1	3
17	0	0	46	2	1	2
29	1	1	36	3	0	0

35. A merchant has 3 P. partly filled with wine; No. 1, contains 1 hhd. 25 gal. 3 qt.; No. 2, 27 gal. 1 qt. 2 gl.; No. 3, 1 hhd. 2 qt. 2 pt. How much wine in the 3 pipes?

Ans. 1 P. 53 gal. 3 qt. 2 gl.

36. A merchant imports in a vessel 56 P. of brandy; 27 hhd. 49 gal. 2 qt. of wine; and 25 gal. 1 qt. 1 pt. 3 gl. of porter. How much in the whole?

Ans. 21 T. 11 gal. 3 qt. 1 pt. 3 gl.

ALE OR BEER MEASURE.

2 Pints,	*pt.*	make	1 Quart,	*qt.*
4 Quarts,		"	1 Gallon,	*gal.*
36 Gallons,		"	1 Barrel,	*bar.*
54 Gallons,		"	1 Hogshead,	*hhd.*

Ale, beer, and milk, are measured by this measure.

37. A milkman sells to one gentleman 5 gal. 3 qt. 1 pt. of milk; another, 11 gal. 1 pt.; and to a third, 9 gal. 2 qt.; how much milk does he sell?

Ans. 26 gal. 2 qt.

DRY MEASURE.

2 Pints,	*pt.*	make	1 Quart,	*qt.*
8 Quarts,		"	1 Peck,	*pk.*
4 Pecks,		"	1 Bushel,	*bus.*
36 Bushels,		"	1 Chaldron,	*ch.*

Grain, fruit, seeds, roots, salt, sand, oysters, &c. are measured by this measure.

38. A farmer raised in one field 4 ch. 3 pk. 1 pt. *of corn; 27* bus. 7 qt. of wheat; 3 ch. 5 bus. 2 pk.

of rye; 7 ch. 4 qt. 1 pt. of oats; 1 ch. 5 bus. 3 pt. of barley; 7 bus. 1 pk. 1 pt. of peas. How much did the field produce?

Ans. 16 ch. 9 bus. 3 pk. 6 qt.

39. A trader has on hand 17 bus. 3 pk. 1 pt. of wheat; 8 bus. 2 pk. 7 qt. of rye; 5 bus. 1 pk. 5 qt. 1 pt. of corn; and 3 bus. 3 qt. of oats; how much in the whole? Ans. 35 bushels.

* * *

MULTIPLICATION OF COMPOUND NUMBERS.

When the sum of two or more numbers of different denominations, and one of the numbers represents a different quantity from either of the others is to be expressed, we find that sum by *adding*; but when several equal numbers of different denominations, are to be collected into one number, the operation is performed by *multiplying*.

The only difference which exists between multiplying a compound and a simple number, is, that in multiplying a *simple* number, we always add 1 to the next left hand place for every 10; but in multiplying a *compound* number, we carry for different numbers from one denomination to another, but for 10, when multiplying one denomination, when that denomination contains more than one column of figures. But in both cases we increase the numbers towards the left, by the same general principle, for in both, we carry for a number in the place we are multiplying, which is equal to one in the next place.

When the multiplier does not exceed 12.

Rule.—Write the multiplier under the last denomination of the multiplicand. Multiply each denomination as in simple numbers, beginning with the least; observing to carry one to the product of the next higher denomination, for as many as it takes of the denomination multiplied, to equal 1 in the next higher.

Question 1. What is the product of 4 £. 6 s. 5 d. 3 qr., multiplied by 3 ?

	Operation 1.				Operation 2.				Operation 3.			
	£.	s.	d.	qr.	£.	s.	d.	qr.	£.	s.	d.	qr.
	4	6	5	3	4	6	5	3	4	6	5	3
				3				3	4	6	5	3
									4	6	5	3
Ans.	12	19	5	1	12	18	15	9				

Illustration.—It was proved in the multiplication of simple numbers, that the multiplicand is placed in the product as many times as the multiplier contains an unit, the additions being performed in the mind. In the multiplication of compound numbers, we obtain a similar result; for every denomination in the multiplicand is a simple number, standing by itself; and by multiplying each denomination by the same multiplier, we place each denomination of the multiplicand in its own product, as many times as there are units in the multiplier; consequently the products of all the denominations will make a compound product, containing the compound multiplicand as many times as the multiplier contain an unit.

In the 3d operation of the last Question, by setting down the multiplicand 3 times, and adding up the several denominations, we obtain a compound number which contains all the denominations of the multiplicand 3 times. In the second operation, when we say, 3 times 3 are 9, 3 times 5 are 15, 3 times 6 are 18, and 3 times 4 are 12, we obtain the same numbers; for we add each denomination together 3 times in the mind. In the 1st operation, we multiply the same as in the 2d, but as there are 2 pence in the 9 farthings and one farthing over, we set down the 1 farthing, and add the 2 pence to the product of the 5, which makes 17 pence. 12 pence being equal to a shilling, by taking 12 from the 17 pence and adding 1 shilling to the product of shillings, and setting down 5, the remaining pence, we obtain the same amount of money as we did by the other operations.

Note 1.—From this illustration, it appears that the price of one unit of any quantity should be multiplied by the whole number of units in the quantity, to obtain the price of the whole. It is equally plain, that when the whole weight of several articles, or the whole amount of several things of any kind, is required, the weight, or contents of one, multiplied by the whole number, will give the weight or contents of the whole.

2. Sold 5 barrels of flour for 1 £. 10 s. 6 d. 3 qr. a barrel; what was the price of the whole?
Ans. 7 £. 12 s. 9 d. 3 qr.

3. What will 6 gal. of rum come to at 6 s. 3 d per gal? Ans. 1 £. 17 s. 6 d.

TROY WEIGHT.

4. A silversmith has 7 tea-pots, each weighing 1 lb. 3 oz. 13 pwt. 11 gr.; what do they all weigh?
Ans. 9 £. 1 oz. 14 pwt. 5 gr.

5. What is the weight of 8 spoons, each weighing 1 oz. 6 pwt. 3 gr.? Ans. 10 oz. 9 pwt.

APOTHECARIES WEIGHT.

6. An apothecary wishes to mix 4 quantities of medicine, each weighing 5 lb. 11 oz. 6 dr. 2 sc. 9 gr. What is the weight of the whole?
Ans. 23 lb. 11 oz. 3 dr. 16 gr.

AVOIRDUPOIS WEIGHT.

7. If a load of hay weigh 1 T. 2 qr. 26 lb. 12 oz. what is the weight of 7 such loads?
Ans. 7 T. 5 cwt. 19 lb. 4 oz.

8. If a merchant sells 9 hhd. of sugar, each weighing 7 cwt. 1 qr. 5 lb. 10 oz.; what is the whole?
Ans. 3 T. 5 cwt. 2 qr. 22 lb. 10 oz.

NEAT WEIGHT.

9. If a certain team can haul 1 T. 1 cwt. 3

qr. 24 lb. of plaster at one load, how much can it haul at 5 loads ?

Ans. 5 T. 9 cwt. 3 qr. 20 lb.

10. What is the weight of 10 bars of iron, each weighing 1 cwt. 20 lb. 2 oz ?

Ans. 12 cwt. 1 lb. 4 oz.

LONG MEASURE.

11. If one side of a square field be 2 fur. 16 ft. in length, what length of fence will it require to inclose said field ? Ans. 1 M. 3 r. 14$\frac{1}{2}$ ft.

12. If a man travels 35 M. 7 fur. 3 r. 10 ft. in one day, how far would he travel in 11 days ?

Ans. 394 M. 5 fur. 39 r. 11 ft.

SQUARE MEASURE.

13. If 8 lots of land measure each 110 A. 3 R. 21 r. 225 ft. 30 in., how much land in the 8 lots ?

Ans. 887 A. 14 r. 167$\frac{2}{4}$ ft. 96 in.

14. How many yards of carpeting, a yard wide, will it require to cover 12 rooms, each measuring 28 yd. 4 ft. Ans. 341 yd. 3 sq. ft.

Note 2.—In painting, paving, plastering and carpeting, yards are considered the highest denomination.

DEFINITIONS.

A Composite Number is a number which is equal to the product of two other numbers ; thus, 24 is the composite number of 6 into 4, or 12 into 2, or 8 into 3.

Component Parts are the numbers multiplied together to produce a composite number ; thus, 6 and 4, 12 and 2, 8 and 3, are the component parts of 24.

When the multiplier exceeds 12, *it is generally more convenient to multiply by two numbers.*

RULE.—Find two numbers whose product is equal to the multiplier. Multiply by one of the numbers, and the product by the other, the last product will be the answer. If the multiplier be not a composite number, find the product of the greatest composite num- in the multiplier, and add the product of the difference between the composite number and the multiplier, into the multiplicand.

SOLID MEASURE.

15. If one pile of wood measure 4 C. 76 ft. 1112 in., how much wood in 17 such piles?

Operation.

C.	*ft.*	*in.*	
4	76	1112	
		4	
18	50	992	contents of 4 piles.
		4	
73	74	512	contents of 16 piles
4	76	1112	contents in 1 pile.
78 C.	22 ft.	1624 in.	contents of 17 piles.

Illustration.—By the reasoning in the illustration of the first rule, 18 C. 50 ft. 992 in, is what 4 piles contain. By making the product of 4 piles the multiplicand, we place the contents of 4 piles in the next product once for every unit there is in the multiplier, consequently we place the contents of 4 piles, 4 times in the next product; and as 4 times 4 are 16, 73 C. 74 ft. 512 in. must be the contents of 16 piles; and adding the contents of 1 pile to this product we obtain the contents of 17 piles. This method of multiplying will hold true with any other numbers, for in all cases the product of the first figure added together as many times as there are units in the 2d multiplier, must give the product of the whole multiplier, when the product of the two numbers by which we multiply is equal to the given multiplier.

16. How much timber in 26 round sticks, each measuring 1 T. 30 ft. 101 in?

Ans. 45 T. 21 ft. 898 in.

17. How much timber in 35 square sticks, each measuring 1 T. 20 ft? Ans. 49 T.

TIME.

18. What is the product of 15 Y. 11 mo. 3 w. 5 d. 2 h. 40 m. 50 s., multiplied by 21?

Ans. 334 Y. 3 mo. 2 w. 2 d. 8 h. 17 m. 30 s.

CIRCULAR MEASURE.

19. If the moon pass through 12°26″ of its orbit in 1 day, how many degrees does it pass through in 15 days? Ans. 180°6′30″.

CLOTH MEASURE.

20. How many yards of cloth in 45 pieces, each measuring 33 yd. 1 na.?

Ans. 1487 yd. 3 qr. 1 na.

21. How many ells English in 53 pieces of sattinet, each 22 E. E. 2 qr. 3 na.?

Ans. 1195 E. E. 3 na.

22. In 60 pieces of velvet, each 21 ells French 2 nails, how many ells? Ans. 1265.

23. In 76 pieces of lace, each 33 E. Fl. 1 qr. 1 na., how many ells?

Ans. 2539 E. Fl. 2 qr.

WINE MEASURE.

24. How much rum in 84 casks, each containing 108 gal. 1 pt. 3 gl.?

Ans. 36 T. 18 gal. 1 qt. 1 pt.

Note 3.—If the multiplier be several hundreds and a part of another hundred, it will be most convenient to find the product of one hundred, and multiply that by the number of hundreds. The product of the remaining part must be added to the product of the hundreds.

ALE AND BEER MEASURE.

25. What is the product of 40 gal. 3 qt. 1 pt., multiplied by 325? Ans. 246 hhd. 1 qt. 1 pt.

DRY MEASURE.

26. If a field of wheat containing 20 acres, produce 16 bus. 1 pt. of wheat to the acre, how much does the whole field produce?

Ans. { 320 bus. 1 pk. 2 qt. or
8 ch. 32 bus. 1 pk. 2 qt.

26. How many bushels of oats must I pay for the rent of a farm of 101 acres, if I pay 3 bus. 7 qts. an acre? Ans. 325 bus. 3 qt.

When the multiplier is a compound number, and the denominations in the multiplicand and multiplier increase by the same number.

It is not often necessary to multiply by a compound number, unless it be in finding the contents of those quantities whose denominations are feet, inches, and parts of an inch.

The inch is variously divided;—sometimes into 8 equal parts; sometimes into 10, and sometimes 12. It will here be considered as divided into 12 equal parts.

TABLE.

12 Fourths,	''''	make	1 Third,	'''.
12 Thirds		"	1 Second,	''.
12 Seconds		"	1 Inch,	*in.*
12 Inches		"	1 Foot,	*ft.*

RULE.

Set the multiplier under the multiplicand, so that every denomination may stand under that of its own name.

Multiply all the denominations in the multiplicand by the ft. in the multiplier, if any, beginning with the least denomination. Then multiply by the inches and seconds, in the same order.

Each separate product must be removed as many places to the right of the figure multiplied, as the figure multiplied by is places from the denomination of ft.; the sum of the several products will be the answer.

As the different denominations increase by 12, we must carry for that number both in multiplying and adding, except when we multiply ft. by ft. where we carry for 10, ft. being the highest denomination.

27. What is the product of 2 ft. 4 in. 2", multiplied by 2 ft. 3 in. 8"?

Operation.

	ft.	*in.*	"	'''	''''
	2	4	2		
	2	3	8		
	4	8	4		
		7	0	6	
		1	6	9	4
Ans.	5	4	11	3	4

Illustration.—In this method of multiplying, a foot is considered the unit, and the inches and seconds are parts of the foot, or of the unit. By placing the product of each denomination under itself, when we multiply by ft., we place the whole multiplicand in the product as many times as the number of ft. containing an unit. As inches and seconds are only parts of a foot, or the unit, we can only place such a part of the multiplicand in the product for each inch, or each second, as 1 in., or 1", is a part of a foot. As an inch is 1 twelfth part of a foot, by removing each product when we multiply by inches, one place to the right hand, the whole product of the inches represents but 1 twelfth part as much as it would, if it had been set directly under the multiplicand, because it takes 12 in every column except the left hand one, to equal 1 in the next left hand denomination. One second being the 12th part of an inch, by removing the product of the second, one place further to the right than we did the product of the inches, it can represent only 1 twelfth part as much as it would, had it stood under the product of the inches.

Note 4.—It may be observed, that when inches, seconds, &c. are multiplied by inches, seconds, &c. without being in connection with any other denomination, the products will be of the same name as the numbers multiplied.

28. What is the product of 29 ft. 9 in. 1″, multiplied by 3 ft. 2 in.?

Ans. 94 ft. 2 in. 9″ 2‴.

29. Multiply 21 ft. 0 in. 5″ by 9 in. 3″.

Prod. 16 ft. 2 in. 6″ 10‴ 3⁗.

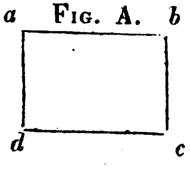
30. Multiply 21 ft. 5″ by 9 ft. 3″.

Prod. 189 ft. 9 in. 1‴ 3⁗.

A surface is a figure having length and breadth, but no thickness:

The outside of all bodies, and the inside of hollow bodies, are surfaces.

FIG. A.



In figure A, the space included by the 4 lines *a b*, *b c*, *c d*, and *d a*, is a surface bounded by said lines.

Note 5.—The contents of all plane or flat surfaces, are found by multiplying the length by the average breadth.

FIG. B.

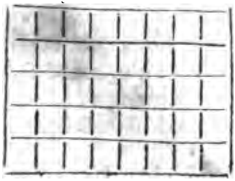


Illustration of the last note.—In figure B, were we to set down 8, the number of small squares on one side, as many times as there are rows of small squares in the figure, and add them together, we should obtain the whole number of small squares in the figure; but as we place the multiplicand in the product once for every unit in the multiplier, we shall obtain the same number, if we multiply 8, the number of squares in one row, by 5, the number of rows; therefore, multiplying the length of any surface by the breadth, will give the contents.

Every surface is supposed divided into squares of that denomination by which the length and breadth of such surface are measured; if measured by yards, into square yards; if by feet, into square feet; if by inches, into square inches, &c.

31. How many square inches in a board 11 in. long and 7 in. wide? Ans. 77.

32. How many square feet in a board 23 ft. long, and 3 ft. wide? Ans. 69.

33. How many feet in a board 20 ft. 7 in. long, 2 ft. 5 in. wide? Ans. 49 ft. 8 in. 11 ″.

34. How many feet in a board 19 ft. long, 2 ft. 9 in. wide? Ans. 52 ft. 3 in.

35. How many feet in a board 18 feet in length, and 10 in. in width? Ans. 15 ft.

36. How many feet in 12 boards, each 14 ft. 1 in. long, 1 ft. 2 in. wide? Ans. 197 ft. 2 in.

37. How many feet in a floor 16 ft. 4 in. long, 15 ft. 4 in. wide? Ans. 250 ft. 5 in. 4 ″.

38. How many feet of boards will it require to lay a floor 17 ft. long, and 16 ft. 11 in. wide?
Ans. 287 ft. 7 in.

Note 6.—Painting and several other kinds of work are reckoned by the square yard; 9 square ft. being equal to 1 square yd. As many times as we can take 9 from the number of square feet, there will be so many square yards.

39. How many square yards in the sides and ends of a room, 15 ft. 7 in. long, 13 ft. 1 in. wide, and 8 ft. high, allowing 5 square yards for the windows?
Ans. 45 yd. 8 ft. 8 in.

*To find the contents of timber, wood and other solid bodies, where the length in all parts is equal, and also the breadth and thickness.**

RULE.

Multiply the length by the breadth, and the product by the thickness.

40. How many solid feet in a square stick of timber, 16 ft. long, 4 ft. wide, 3 ft. thick?

Operation.

```
 ft.
 1 6
   4
 ———
 6 4
   3
 ———
1 9 2
```

Illustration.—Multiplying the length by the breadth gives the number of square feet on one side, as was proved in the last illustration. If the stick were but 1 ft. thick, there would be as many solid, as square ft., for if the stick in that case were sawed into 64 equal blocks, each block would measure 1 ft. on every side, and contain 1 solid foot. But as the given stick is 3 ft. thick, we have what is equal to 3 sticks of the given length and breadth, and 1 ft. thick, consequently there will be 3 times 64 solid ft. in the given stick. The rule is equally true as it respects any other numbers, for, as the number of square ft. or in., &c. into which one side is divided, is equal to the number of solid ft. or in. on one side, when 1 foot, or 1 inch, of the thickness is considered, there will always be as many solid feet or inches for every foot, or inch, of the whole thickness, as there are feet or inches in the thickness; therefore multiplying the number of square feet, or inches, &c. on one side, by the number of feet or inches in the thickness, will give the whole solid feet, or inches, &c. in a solid body of the above form.

---

* When the length of one side is greater than that of the oth— and the variation is a true slope, the average length should be tak and the same of the width and thickness.

41. How many solid feet in a stick of timber 19 ft. long, 1 ft. 3 in. wide, and 1 ft. thick?

Ans. 23 ft. 9 in.

42. How many solid feet in a stick of timber 23 ft. long, 1 ft. 4 in. wide, and 11 inches thick.

Ans. 28 ft. 1 in. 4″.

43. How many solid feet in a stick of timber 30 ft. 6 in. long, 2 ft. wide, and 1 ft. 10 in. thick?

Ans. 111 ft. 10 in.

44. How many solid feet in 7 sticks, each 56 ft. long, 11 in. wide, and 10 inches thick?

Ans. 299 ft. 5 in. 4″.

Note 7.—To find how many tons of timber, or how many cords of wood, are contained in any number of feet, we have only to divide the given number of ft. by the number of solid ft. in a ton or cord.

45. How many tons of hewn timber in 24 sticks, each 72 ft. 8 in. long, 2 ft. wide, and 1 ft. 10 in. thick? Ans. 127 T. 44 ft. 8 in.

46. How much wood in a load 8 ft. long, 4 ft. high, and 4 ft. wide? Ans. 1 cord.

47. How much wood in a load 11 ft. 4 in. long, 3 ft. 7 in. high, and 4 ft. wide?

Ans. 1 C. 34 ft. 5 in. 4″.

16 solid feet are frequently called 1 foot of wood, and 8 feet of wood make 1 cord, because there are 8 times 16 in 128, the number of solid feet in a cord; to find how many feet of wood there are in any number of solid feet, we have only to divide by 16.

The answers in this work will be given in cords and solid feet.

48. How much wood in a load 9 ft. 10 in. long, 4 ft. 9 in. high, and 4 ft. wide, allowing 3 in. in the height for loose package? Ans. 1 C. 49 ft.

49. How many cords in a pile of wood 37 ft. 10 in. long, 5 ft. 3 in. high, and 2 ft. 4 in. wide?

Ans. 3 C. 79 ft. 5 in. 6″.

50. How many solid or cubick feet in a bale of

goods, 6 ft. 2 in. long, 3 ft. 3 in. wide, 2 ft. 8 in. deep? Ans. 53 ft. 5 in. 4″.

Diggers work by the square of 6 ft. long, wide, and deep, which contains 216 solid feet.

51. How many squares of the above dimensions must he dig to make a canal 400 feet long, 30 feet wide, and 4 feet deep? Ans. $222\frac{48}{216}$.

Remark.—When we multiply feet and inches by feet and inches once, the inches in the product do not express square inches, for it has been proved that each inch is a twelfth part of a foot, and there being 144 square inches in a square foot, each inch in the product will represent 12 square inches. When we multiply the 2d time by ft. and in. to get the solid contents, the inches in the last product become twelfths of a solid foot, consequently each inch must represent 144 solid inches, which is the twelfth part of 1728, the number of solid inches in a solid foot.

When we say, a board contains 9 ft. 6 in., the 6 in. represent a part of the board equal to a board 12 in. long and 6 in. wide. So in a stick of timber measuring 9 ft. 6 in., the 6 in. represent a part of the stick equal to a stick 12 in. long, 12 in. wide, and 6 in. thick.

Questions to be performed by the addition and multiplication of compound numbers.

1. A teamster loaded his waggon with 1 hhd. of molasses weighing 9 cwt. 1 qr. 21 lb.; 2 bales of cotton, each 1 cwt. 3 qr. 6 oz.; 3 bar. of flour, each 2 cwt. 1 lb.; and 4 bars of iron, each 1 cwt. 3 lb. What was the weight of the load?

Ans. 1 T. 3 cwt. 8 lb. 12 oz.

2. What is the whole weight of a ship's cargo consisting of 103 hhd. of salt, each weighing 4 cwt. 1 qr. 21 lb.; 35 hhd. of wine, each 4 cwt. 3 qr. 2 lb.; 106 boxes of lemons, each 2 qr. 16 lb.; 126 casks of

raisins, each 2 qr. 16 lb. 10 oz.; and other articles to the amount of 65 T.?

Ans. 103 T. 13 cwt. 3 qr. 3 lb. 12 oz.

3. How many feet of boards will it require to cover a building, 60 ft. 6 in. long, 40 ft. 3 in. wide, 22 ft. high, and each side of the roof 24 ft. 2 in.; allowing 523 ft. 3 in. for the gable ends, and making no deduction for doors or windows?

Ans. 7880 ft. 5 in.

* * *

SECTION V.

SUBTRACTION OF COMPOUND NUMBERS.

Subtraction of compound numbers is taking one compound number from another, or a part of a compound number from itself, to find the difference.

Rule 1.—Write the least number under the greater in such a manner that every denomination may stand under one of its own name.

Commence subtracting with the least denomination, and set the difference between the two numbers in each denomination in a line below.

ENGLISH MONEY.

Question 1. What is the difference between 35 £. 15 s. 8 d. 3 qr., and 20 £. 8 s. 3 d. 1 qr.?

Operation.

	£.	*s.*	*d.*	*qr.*
	35	15	8	3
	20	8	3	1
Ans.	15	7	5	2

Illustration.—The 15 £. is the difference between the pounds in the minuend and the pounds in the subtrahend; 7 s. is the difference of the shillings, 5 d. the difference of the pence, and the 2 qr. the difference of the farthings; therefore the compound number, 15 £. 7 s. 5 d. 2 qr. is the difference between the compound numbers given. As each denomination by itself is a simple number, when the under numbers in all the denominations are the smallest, subtracting by the above method will give the difference between any numbers, for it is only subtracting as many simple numbers as there are denominations.

2. If 99 £. 1 s. 10 d. 1 qr. be taken from 100 £. 19 s. 11 d. 3 qr., what will remain?

Ans. 1 £. 18 s. 1 d. 2 qr.

3. Required the difference between 19 s. 3 d. 2 qr. and 2 d. 1 qr. Ans. 19 s. 1 d. 1 qr.

TROY WEIGHT.

4. From 9 lb. 11 oz. 19 pwt. 23 gr. take 6 lb. 1 oz. 17 pwt. 14 gr.

	lb.	*oz.*	*pwt.*	*gr.*
	9	11	19	23
	6	1	17	14
Rem.	3	10	2	9

5. How much heavier is a gold necklace, which weighs 5 pwt. 19 grs. than one which weighs 4 pwt. 1 gr? Ans. 1 pwt. 18 grs.

APOTHECARIES WEIGHT.

6. Subtract 5 lb. 7 oz. 6 dr. 19 grs. from 11 lb. 10 oz. 7 dr. 2 sc. 19 grs.

(7.)

lb.	*oz.*	*dr.*	*sc.*	*grs.*
11	10	7	2	19
5	7	6	0	19
6	3	1	2	0

7. Take 1 lb. 6 oz. 2 grs. from 1 lb. 7 oz. 3 dr. 1 sc. 17 grs? Ans. 1 oz. 3 dr. 1 sc. 15 grs.

Rule 2.—When the under number in any denomination is greater than the upper one, add as many units to the upper one as it takes of that denomination to make 1 in the next greater, and subtract the under number from that sum; then add 1 to the lower number in the next higher denomination.

AVOIRDUPOIS WEIGHT.

8. Required the difference between 18 T. 3 qr. 19 lb. 12 oz. 5 dr. and 12 T. 14 cwt. 22 lb. 3 oz. 11 dr.

T.	*cwt.*	*qr.*	*lb.*	*oz.*	*dr.*
18	00	3	19	12	5
12	14	0	22	3	11
5	6	2	25	8	10

Explanation.—We cannot subtract 11 dr. from 5 dr., we therefore add 16 dr. (which are equal to 1 in the denomination of oz.) to the 5 drs. and subtract 11 from 21, the sum of 16 and 5, and add 1 to the 3 in the next place. In the denomination of lb., because 28 lb. equal 1 qr., we add 28 to 19, and subtract 22 from 47, the sum. In the next denomination, the under place being occupied by a cipher, we suppose 1 qr. placed where the cipher is, and subtract it from the 3 qr. to balance the 28 lb., which we added to the 19 lb. As there are no cwt. in the minuend, we suppose 1 T., that is, 20 cwt., to stand in the place of cwt., and subtract the 14 from it, and then add 1 T. to the 12 T.

9. Bought 12 cwt. 2 qr. of fish; how much remained after selling 9 cwt. 27 lb.? Ans. 3 cwt. 1 qr. 1 lb.

NEAT WEIGHT.

10. From 3 T. 2 cwt. 1 qr. 21 lb. 13 oz. take 17 cwt. 11 lb. 3 oz. 7 dr?

Rem. 2 T. 5 cwt. 1 qr. 10 lb. 9 oz. 9 dr.

11. If a load of hay together with a waggon, weigh 2 T. and the waggon weigh 15 cwt. 3 qr. 11 lb., how much hay? Ans. 1 T. 4 cwt. 14 lb.

LONG MEASURE.

12. If A and B travel to the city of Washington from the same place, and the road on which A travels, be 24 m. 8 r. 16 ft. 4 in., and that which B travels, be 21 m. 3 fur. 38 r. 14 ft. 6 in., how much farther does A travel than B?

Operation 1.					Operation 2.				
M.	*fur.*	*r.*	*ft.*	*in.*	*M.*	*fur.*	*r.*	*ft.*	*in.*
24	0	8	16	4	23	7	48	15	16
21	3	38	14	6	21	3	38	14	6
2	4	10	1	10	2	4	10	1	10

Illustration.—When the upper number in any denomination is less than the one below it, by adding as many to the upper number as it takes of that particular denomination to make 1 in the next higher, and subtracting the under number from the sum, we subtract the under number from the number above it and 1 taken from the upper number in the next higher denomination. Now if we were to take away 1 from the upper number in the next higher denomination, and subtract the under number from the remainder, we should certainly obtain the difference between the under and upper numbers of both denominations; but as we do not generally take 1 from the upper number, we may suppose that we have added 1 to the upper number, after we suppose 1 carried from it to the other denomination, and by adding 1 to the under number, there will be no difference between the 1 added to the upper and the 1 added to the under; consequently the difference must be between the two numbers as they were given, after supposing 1 taken from the upper number, and carried to the next right hand denomination.*

An example will make this principle plain. In operation 1, of the last question, we subtract in the common method, supposing 12 in. added to the 4 in., 40 r. to the 8 r., and 8 fur. set in the place of the cipher because these numbers are just equal to 1 in the next higher denomination, and carrying 1 in each instance to the next higher denomination.

In operation 2, instead of supposing a number added to the upper number, when it is the smallest, we actually separate the numbers. As we cannot take 6 from 4 in. we take 1 foot from the 16 ft. and add 12 in. (which are equal to 1 ft.) to the 4, and subtract the 6 from the 16. As we have taken 1 ft. from the 16, we shall have more to add to the 14.

By examining operation 1, it may not at first appear from what place we are to obtain the 40 r. which are to be added to the 8, as there are no fur. in the minuend. These really come from 1 of the miles in the 24, as will be seen in operation 2. We take 1 m. from the 24, which is equal to 8 fur., and set 7 of the fur. in the place of furlongs, and for the other furlong, we add 40 r. to the 8 r., which makes all the upper numbers the largest. Had we not actually taken 1 of the miles from the 24, we should have supposed 40 r. added to the 8, and 1 fur. added to the 3 fur.; in that case, we should have supposed 8 fur. added in the place of fur., and 1 m. added to the 21, to balance the 8 fur. By the 1st operation, we suppose 9 fur. added to the minuend, and 1 m. and 1 fur. added to the subtrahend, but there being no difference between 1 m. and 1 fur., and 9 fur. the remainder of miles, furlongs and rods, must be the difference between the given numbers of those denominations.

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* Instead of adding 1 to the next under number, it would be equally correct to take 1 from the upper, but it is not always so convenient.

The whole minuend in operation 2, is equal to the minuend in the 1st operation, for what we took from the miles and feet, we added to the other denominations, and as we find the same answer by both operations, it appears that we do essentially the same thing whether we *suppose* numbers added, or actually remove them from higher to lower denominations in the minuend.

13. Two roads leading to the same place, measure forty-five miles, six furlongs and thirteen feet; the shortest one is nineteen miles, two furlongs and fifteen feet; what is the length of the longest road, and how much longer is one than the other?

Ans. { Longest road, 26 m. 3 fur. 39 r. 14¼ ft.
Difference, 7 m. 1 fur. 38 r. 16 ft.

Note 1. When one of two numbers, which are to be added together, contains a fraction, the fraction must be set at the right hand of the sum of the other numbers. When one number is to be taken from another, which contains a fraction, the fraction must be brought down without being altered.

14. How much greater is the circumference of a circle than the diameter, if the former be 16 ft. and the latter 5 ft. 1 in.? Ans. 10 ft. 11 in.

15. Two miles and five rods of fence will enclose the whole of a certain farm, and one mile, one furlong and ten feet, one half of it. How much more fence will be required to enclose the whole than one half? Ans. 7 fur. 4 r. 6½ ft.

SQUARE MEASURE.

16. How much more land in a field, which contains 25 A. 3 R. 37 r. 160 ft. 143 in., than in another, which contains 17 A. 179 ft.?

	A.	*R.*	*r.*	*ft.*	*in.*
	25	3	37	160	143
	17	0	00	179	000
Ans.	8	3	36	253¼	143

17. Subtract 1 A. 1 R. 1 r. 1 ft. 1 in. from 2 A.
Rem. 2 R. 38 r. 270¼ ft. 143 in.

18. A pond measuring fifteen acres, one hundred and fifteen feet, is situated in the middle of a farm, containing one hundred acres and fourteen rods. How much land in said farm exclusive of that occupied by the pond? Ans. 85 A. 13 r. 157¼ ft.

SOLID MEASURE.

19. If a man sell 6 C. 127 ft. 1006 in. from a pile of wood measuring 11 C. 120 ft. 1009 in., how much will remain?

	C.	ft.	in.
	11	120	1009
	6	127	1006
Ans.	4	121	3

20. A man agrees to chop 15 C. 11 ft. of wood; but doing his work better than was expected, his employer pays him for chopping 17 C.; for how much is he paid, that he did not chop?
Ans. 1 C. 117 ft.

21. A man cut 49 ft. of timber from a square stick, which measured 2 T. 6 ft.; how much remained? Ans. 1 T. 7 ft.

22. How many more solid feet in a cask, which measures 43 ft., than in one measuring 42 ft. 75 in.?
Ans. 1653 in.

TIME.

23. From 131 Y. 51 s., subtract 65 Y. 2 mo. 3 w. 4 d. 20 h. 40 m.

Y.	mo.	w.	d.	h.	m.	s.
131	00	0	0	00	00	51
65	2	3	4	20	40	00
65	10	0	2	3	20	51

24. From 15 Y. subtract 3 w. 21 h. 25 s.
Rem. 14 Y. 12 mo. 6 d. 2 h. 59 m. 35 s.

25. A note was given in 1822, April 21, and paid 1826, March 15; how long was it standing?

Operation.

	Y.	*mo.*	*d.*
	1826	3	15
	1822	4	21
Ans.	3	10	24

Explanation.—In this example, 30 days are reckoned a month: this is not strictly true in all cases, but sufficiently near for most practical purposes. When 30 days are reckoned a month, we must carry for the number of Calendar months in a year, which is 12.

26. How long was that note on interest, which was given in 1821, December 29, and paid 1824, June 22? Ans. 2 Y. 5 mo. 23 d.

Note 2. In the place of months, set down the number of any given month, counting from the beginning of the year, calling January 1, February 2, March 3, &c.

27. If a note were given on interest, 1826, January 3, and paid August 1, of the same year; how long was said note on interest? Ans. 6 mo. 28 d.

PROOF.

Add the remainder and subtrahend together, if the question has been correctly performed, the sum will equal the remainder, for the difference of two numbers added to the less, will give a sum equal to the greater.

CIRCULAR MEASURE.

28. If 3 cir. 2 s. 12° 57′ 48″, be taken from 5 cir. 1 s. 29° 40′ 51″, what will remain?

cir.	*s.*	°	′	″
5	1	29	40	51
3	2	12	57	48

29. After the earth has passed through 1 s. 15 ° 22 ′ of its orbit, what part of its orbit remains for it to pass through to complete a revolution?

Ans. 10 s. 14 ° 38 ′.

CLOTH MEASURE.

30. How much longer is a piece of cloth, which measures 37 yd. 1 qr. 2 na., than one which measures 19 yd. 3 qr. 3 na.? Ans. 17 yd. 1 qr. 3 na.

31. A merchant sells 3 yd. 1 qr. from a piece of cloth measuring 30 yd. 3 na., how much remains?

Ans. 26 yd. 3 qr. 3 na.

32. If a pattern for a coat which requires 1 E. E. 4 qr., be taken from a piece of broadcloth measuring 21 E. E. 2 qr., how much will remain?

Ans. 19 E. E. 3 qr.

WINE MEASURE.

33. How much more wine in a cask which contains 73 gal. 1 qt. 3 gl., than in another which contains 51 gal. 1 pt. 1 gl.? Ans. 22 gal. 1 pt. 2 gl.

34. A merchant after selling a certain quantity of rum from a hogshead, finds there are 45 gal. 2 qt. 1 pt. remaining. How much did he sell?

Ans. 17 gal. 1 qt. 1 pt.

ALE OR BEER MEASURE.

35. If 37 gal. 1 pt. of beer be taken from a cask containing 49 gal., how much will remain?

Ans. 11 gal. 3 qt. 1 pt.

DRY MEASURE.

36. If 18 bus. 1 pk. 7 qt. of grain be taken from a bin containing 57 bus., how much will remain?

Ans. 38 bus. 2 pk. 1 qt.

37. A farmer after selling grain from a bin containing 57 bus. has 38 bus. 2 pk. 1 qt. left; how much did he sell?

Ans. 18 bus. 1 pk. 7 qt.

Questions to be performed by Addition and Subtraction of compound numbers.

1. A merchant imports 75 T. 1 qr. 20 lb. of iron in one vessel, and 60 T. 15 cwt. 3 qr. in another; he then sells 10 T. 3 qr., how much has he left?
Ans. 125 T. 15 cwt. 1 qr. 20 lb.

2. A father's age is 75 Y. 6 mo. 3 w.; his eldest son's 50 Y. 5 d., and his youngest son's 25 Y. 15 h. If the difference between the ages of the sons be added to the age of the father, what will be the sum of the time?
Ans. 100 Y. 6 mo. 3 w. 4 d. 9 h.

Questions to be performed by Multiplication and Subtraction of compound numbers.

1. A gentleman after selling 27 A. 1 R. 14 r. from a farm containing 100 A. 27 ft., has three other farms each equal to what he has left of that from which he sells a part; how much land has he?
Ans. 290 A. 2 R. 24 r. 108 ft.

2. A boy was sent to a store with 16 £. 9 s. 1 d. 3 qr., to pay a debt of 15 £. 9 d. 2 qr. After paying the debt, he takes up goods to 5 times the amount of what money he had left, and leaves the remainder of his money in part pay. What was the new debt?
Ans. 5 £. 13 s. 5 d.

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DIVISION OF COMPOUND NUMBERS.

Division of compound numbers is ascertaining how many times a simple number can be taken from the several denominations of a compound number; or it is separating a compound number into several parts.

As the number to be divided is composed of several denominations, the quotient will generally consist of different denominations.

RULE 1.

Divide each denomination of the dividend by the divisor, beginning with the highest, the same as in simple numbers.

1. Divide 15 £. 12 s. 9 d. 3 qr. by 3.

Operation.

	£.	s.	d.	qr.	
3)	15	12	9	3	
	5	4	3	1	quotient.

2. If 4 yd. of cloth cost 16 £. 12 s. 8 d., what is the cost of one yard ?

	£.	s.	d.
4)	16	12	8
	4	3	2

Illustration of Rule 1,—In the division of simple numbers, 1 is placed in the quotient every time we take the divisor from the dividend. By rule 1, in the division of compound numbers, we divide each denomination as a simple number, consequently, the quotient of each denomination contains 1 as many times as the divisor can be taken from that particular denomination. In the operation of the last question, we divide the price of 4 yards by 4. By taking as many units of any denomination as there are units in the divisor, we take 1 unit from the dividend for each unit in the divisor ; but, as we place but 1 unit in the quotient every time we take the whole divisor from the dividend, thus, 1 set in the quotient, answers to any one in the divisor. 16 £. 12 s. 8 d. is the price of 4 yards, but as we put but 1 in the quotient of each denomination every time we take 4 from that denomination, the whole quotient can only be a 4th part of the whole dividend; consequently, 4 £. 3 s. 2 d. is the price of 1 of the 4 yards.

3. If 9 pieces of cloth cost 72 £. 18 s. 9 d. what is the cost of 1 piece? Ans. 8 £. 2 s. 1 d.

4. If 7 equal loads of hay weigh 7 T. 14 cwt. 21 lb. what does 1 load weigh?
Ans. 1 T. 2 cwt. 3 lb.

5. What is the 8th part of 32 A. 40 r. 160 ft.?
Ans. 4 A. 5 r. 20 ft.

RULE 2.

If in dividing any denomination except the least there be a remainder, multiply it by as many of the next less denomination as it takes to make 1 in that denomination from which the remainder is derived, and add the number in the less denomination to the product. Then divide the sum thus found by the divisor.

6. If 8 bars of iron weigh 18 cwt. 3 qr. 20 lb. 8 oz. neat, what is the weight of 1 bar?

Operation.

cwt. *qr.* *lb.* *oz.*
18 3 20 8

8) 18 (2 quotient of 16 cwt.
16

2 remainder.
4 multiply by 4 the No. of qr. in 1 cwt. and the 3 qr. added.

8) 11 (1 quotient of 8 qr.
8

3 remainder of the quarters.
25 multiply by 25 and add the 20 lb.

75
20

8) 95 (11 quotient of 88 lb.
8
—
15
8
—
7 remainder of lbs.
16 multiply by 16 and add 8 oz.
—
8) 120 (15 quotient of the oz.
8
—
40
40
—

Whole quotient 2 cwt. 1 qr. 11 lb. 15 oz.

Illustration of Rule 2.—When a remainder occurs, it is always less than the divisor, so that the divisor cannot be taken from it, and another unit of that denomination cannot be set in the quotient. Yet there being a remainder, the number of units put in the quotient is not the exact quotient, and although we cannot have another unit in the larger denomination, we may have 1 or more units in the less denomination, where each unit does not express so much. In the last question, each bar will weigh more than 2 cwt. but not so much as 3. Had the weight of the 8 bars been 16 cwt. the weight of each bar would have been just 2 cwt., and if the weight of the whole had been 24 cwt., the weight of each would have been just 3 cwt.; but as it is more than 16 and less than 24, the weight of 1 bar must be somewhere between 2 and 3 cwt. The 11 qr. express all the quarters in the 2 cwt. which were left and the 3 given quarters; the 95 lb. contain the 3 remaining qr. and the 20 lb. given, and the 120 oz. contain the 7 remaining lb. and the 8 oz. given, for 4 qr. being equal to 1 cwt. by multiplying the 2 cwt. which were left by it, we put 4 into the product for each cwt. remaining, and so of the other denominations.

In dividing the quarters, pounds and ounces, by the same divisor that we did the hundreds, we take the same part of the several denominations, and these denominations have not been divided before, for we only divide the remainders brought into a lower denomination, and the given numbers of each denomination added to them; consequently, 2 cwt. 1 qr. 11 lb. 15 oz. is an eighth part of 18 cwt. 3 qr. 15 lb. 11 oz.

7. A gentleman divided 528 A. 3 R. 9 r. of land, equally between his 7 sons. How much did each receive? Ans. 75 A. 2 R. 7 r.

8. How far must a man travel in one day, to go 351 miles in 10 days? Ans. 35 M. 32 r.

9. Wishing to divide a stick of hewn timber, containing 2 T. 28 ft. 1104 in. into 11 equal parts, how much must each part contain?
Ans. 11 ft. 1200 in.

10. Bought 12 silver spoons, weighing 2 lb. 10 oz. 14 pwt. 12 gr.; what was the weight of each?
Ans. 2 oz. 17 pwt. 21 gr.

11. A silversmith is ordered to make 9 pair of sugar tongs from 8 oz. 18 pwt. 21 gr. of silver. What will be the weight of each pair?
Ans. 19 pwt. 21 gr.

12. A teamster draws a pile of wood consisting of 12 C. 92 ft. at 11 loads. How much does he haul at a load? Ans. 1 C. 20 ft.

13. An army consisting of 10 brigades, consumes 5000 lb. 15 oz. 10 dr. of bread in one day; how much does each brigade consume?
Ans. 500 lb. 1 oz. 9 dr.

RULE 3.

When the divisor is large and a composite number, divide the dividend by one of its component parts, and that quotient by the other.

14. If 24 yards of cotton cloth cost 1 £. 16 s. what is the price of one yard?

£. s. d.
3) 1 16
8) 12
Ans. 1 s. 6 d.

Illustration of Rule 3.—The product of the component parts of a number, is always equal to the number itself, therefore, dividing by one of the component parts, we take a part of the dividend, which is as much less than the whole dividend, as the other component part is less than the whole divisor; and dividing that part of the dividend by the other component number, must give the quotient of the dividend divided by the whole divisor. In the last question, 12 s. is a third part of the dividend, but as 8 is a third part of the whole quantity, 24 yd., the 12 s. must be the price of 8 yd.; and dividing the price of 8 yards by 8, we obtain the price of 1 yard, in the same manner as shown before.

15. A farmer sold 56 bushels of corn for 11 £. 8 s. 8 d.; what did he receive a bushel?

Ans. 4 s. 1 d.

16. If 100 acres produce 2198 bus. 1 pk. 6 qt. of rye, what does one acre produce?

Ans. 21 bus. 3 pk. 7 qt. 1 pt.

17. Suppose the earth to move through the space of 50'159'999'999 M. 6 fur. 20 r. in 84 years; how far does it move in one year?

Ans. 597'142'857 M. 1 fur. 5 r.

Note 1.—If the divisor be hundreds, divide the dividend by the number of hundreds, then proceed as directed by Rule 3.

18. If 600 pounds of copperas cost 2 £. 10 s. what is it per pound?

£. s. d.
6) 2 10

10) 8 4 price of 100 pounds.

10) 10 price of 10 pounds.

1 d. price of 1 pound.

19. 500 soldiers receive 50221 A. 3 R. 20 r. of bounty land; what is the share of each?
Ans. 100 A. 1 R. 31 r.

Note 2.—If the divisor be not a composite number, we must divide the dividend by the whole divisor.

20. If a field containing 19 A. produce 17 T. 3 cwt. 1 qr. 8 lb. of hay, neat, what does one acre produce?

T. *cwt.* *qr.* *lb.*
17 3 1 8
20

19) 343 (18 cwt. 0 qr. 7 lb.
19

153
152

1
4

19) 5 (0
25

19) 133 (7
133

Explanation.—We cannot divide 17 by 19, we therefore multiply the tons by 20, the number of cwt. in a ton, add the 3 cwt. to the product, and find 18 cwt. the first quotient. Neither can we divide the 5 qr. by 19, so that we must bring the quarters to pounds, and then divide.

21. If a planter sell 173 hhd. of tobacco for 3480 £. 18 s. 1 d. what does he receive per hhd?

Ans. 20 £. 2 s. 5 d.

22. If each hhd. in the last question contain 902 pounds, what does the planter receive per pound?

Ans. 5 d. $1\frac{374}{902}$ qr.

23. A stock of 12 boards measures 195 ft. 8 in.; how much does 1 board measure?

Ans. 16 ft. 3 in. 8 s.

24. If it requires 5005 ft. of boards to cover the sides of a square building, how many feet does it take to cover one side? Ans. 1251 ft. 3 in.

25. What is the measure of one square stick of timber, if 13 sticks of equal dimensions, measure 18 T. 24 ft. 365 in. Ans. 1 T. 21 ft. 161 in.

26. What is the 7th part of 19 C. 81 ft. of wood?

Ans. 2 C. 103 ft.

Questions to be performed by Subtraction and Division of compound numbers.

1. What is one-third of the difference between 6 cwt. 21 lb., and 3 cwt. 22 lb., neat?

Ans. 3 qr. 24 lb. 10 oz. $10\frac{2}{3}$ dr.

2. A man dying, left a farm of 200 A. 3 R. 1 r., of which his widow was to receive one-third, and the remainder to be divided equally between 6 children: What was the widow's share; also, what was the share of each child?

Ans. { Widow's share, 66 A. 3 R. 27 r.
Each child's share, 22 A. 1 R. 9 r.

3. 24 men agree to construct 7 M. 1 fur. 24 r. of road in a given time. After completing one-sixth of

the distance, they employ 8 men more. What distance does each man construct before and after the 8 men are employed?

Ans. { Each man constructs 16 r. before, and 1 fur. 20 r. after the 8 men are employed.

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SECTION VI.

REDUCTION OF COMPOUND NUMBERS.

The same quantity may be expressed by different numbers. Thus, 2 weeks or 14 days express the same space of time. It is easy to perceive that if we express the same quantity by different numbers, that each unit in the larger number must represent a less part of the quantity than an unit in the smaller number. The 14 units in the 14 days, denote as much time as the 2 units in the 2 weeks; but a single unit in the 14 days implies but 1 day, whereas a single unit in the 2 weeks denotes 7 days, and is equal to 7 units in the 14 days.

The more parts we divide any thing into, the less each part must be. Let the figure 1 stand for one apple, this figure denotes the whole; but if we divide the apple into 4 equal parts, and have 1 unit to express each part, we must have 4 units to express the same apple which was expressed by 1 unit before it was divided; and if each quarter be divided into 2 equal parts, the whole apple will be divided into 8 equal parts, and the figure 8 will be required to express the whole apple, when each unit is to express 1 part. The figure 1, in one instance, expresses as much as 4 other units, and in the other, as much as 8 units express, for the 4 or the 8 denotes but 1 apple. From what has been said, it appears that we can fix no limit to the value of an unit, for every time we divide any thing, the parts must become smaller, and an unit to express 1 part each time the quantity is divided, will become less in value in the same proportion.

Reduction, is changing a number from one denomination to another, without altering its value.

When a number in one denomination is to be changed to a less denomination.

RULE.

Multiply the given number by as many as it takes of that denomination to which it is to be reduced, to make 1 in the given denomination.

Question 1. In 3 hhd. of rum, how many gallons?

Operation 1.	Operation 2.
3	63
63	63
——	63
189	——
	189

Illustration.—It was proved, pages 43, 44, 45, that we place the multiplier in the product, once for each unit in the multiplicand, consequently, multiplying a greater denomination by as many as it takes of a less to make one in the greater, we place the value of each unit of the greater in the product. The product is equal to the multiplicand, for we place the multiplier, which is equal to an unit in the multiplicand, once in the product for each unit in the multiplicand. The product is of the same denomination as the multiplier, for it is the same number that we should obtain, were we to set down the multiplier as many times as there are units in the multiplicand, and add them together, the sum of several numbers always being of the same denomination as the numbers themselves; or the product is of the same denomination as the multiplier, because it is as much larger than the multiplicand as the multiplier is larger than one.

In the 2d operation of the last question, by setting down the number of gallons in one hogshead, as many times as there are hhds. and adding them together, we obtain a number expressing all the gallons in the 3 hhds. but by multiplying the number of hhds. by 63, the number of gallons contained in 1 hhd., we obtain the same number. The product being equal to the multiplicand, 189 gal. express the same quantity of liquor as the 3 hhds., but in a less denomination.

2. In 25 £. how many shillings? Ans. 500.
3. In 500 s. how many pence? Ans. 6000.
4. In 6000 d. how many farthings? Ans. 24000.
5. In 10 lb. Troy weight, how many ounces? Ans. 120.
6. In 120 oz. how many pennyweights? Ans. 2400.
7. How many drams in 7 oz. Apothecaries' weight? Ans. 56.
8. In 14 T. how many hundreds? Ans. 280.
9. In 280 cwt. how many quarters? Ans. 1120.
10. In 1120 qr. how many pounds? Ans. 31360.
11. In 1120 qr. how many pounds, Neat weight? Ans. 28000.

Note 1. When the given number consists of two denominations, the greater denomination must be reduced as directed by the rule, and if the less denomination be of the same name as the multiplier, it must be added to the product. The sum will express the value of the two given denominations, for the product is equal to the number in the greater denomination, and being of the same denomination as the multiplier, and the number in the less denomination being of the same denomination, by adding it to the product, we obtain both of the given numbers in the sum.

12. In 120 M. 7 fur. how many furlongs?

```
 M.    fur.
1 2 0   7
  8
-------
9 6 0 number of furlongs in 120 M.
    7 given furlongs added.
-------
9 6 7 number of furlongs in 120 M. 7 fur.
```

13. In 15 yd. 3 qr. how many quarters?
Ans. 63.

14. How many quarters in 15 E. Fl. 2 qr.?
Ans. 47.

15. In 15 E. E. 2 qr. how many quarters?
Ans. 77.

16. How many quarters in 15 E. Fr. 1 qr.?
Ans. 91.

17. In 16 Y. 3 mo. how many months?
Ans. 211.

18. In 125 square M. 6 A. how many acres?
Ans. 80006.

When a number is to be reduced to any less denomination.

RULE.

Multiply the highest denomination by as many as it takes of the next less to equal one in the greater; multiply the product by as many as it takes of the next less denomination to make one in that product.

Multiply each succeeding product in the same manner until the given number is reduced to the denomination required, and add to each product the given number of the same denomination.

19. In 6 T. 1 qr. 10 oz. neat, how many ounces?

Operation.

```
    T. cwt. qr. lb. oz.
    6   0    1   0  10
    2 0
    -----
    1 2 0 number of hundreds in 6 T.
      4
    -----
    4 8 0 number of quarters in 6 T.
        1 the given quarter added.
    -----
    4 8 1 the number of quarters in 6 T. 1 qr.
      2 5
    -----
  2 4 0 5
  9 6 2
  -------
1 2 0 2 5 the number of pounds in 6 T. 1 qr.
      1 6
---------
7 2 1 5 0
1 2 0 2 5
-----------
1 9 2 4 0 0 the number of ounces in 6 T. 1 qr.
        1 0 the given ounces added.
-----------
1 9 2 4 1 0 the number of ounces in 6 T. 1 qr. 10 oz.
```

Illustration of the last rule.—By the last illustration, each product expresses the same value as the multiplicand which precedes it, and as we add the several numbers in the less denominations to products of the same name, the last product with the number added to it, (if there be any given of that denomination,) will express the same value as all the numbers given of the several denominations. See operation of the last question.

20. In 21£ 15 s. 11 d. 3 qr., how many farthings?
Ans. 20927.

21. In 51 C. 127 ft., how many solid feet of wood?
Ans. 6655.

22. In 16 lb. 17 pwt. 21 gr. how many grains?
Ans. 92589.

23. How many inches long must a rope be to reach across a yard which is 8 rods across?
Ans. 1584.

24. 6 shillings equal 1 dollar, how many pence in 1000 dollars? Ans. 72000.

25. How many seconds has that person lived whose age is 27 Y. 2 w. 3 d. 25 m.?
Ans. 850'609'500.

26. If 1 square rod of land produce 1 pk. of corn, how many pecks will 5 A. 3 R. produce?
Ans. 920.

27. In 2 T. 6 cwt. 1 qr. 20 lb. 14 oz. 10 dr., how many drams? Ans. 1'331'434.

28. How many seconds in 15°?
Ans. 54'000.

29. How many pints in one hogshead?
Ans. 504.

30. How many pints in a hhd. of beer?
Ans. 432.

31. 28 shillings equal one guinea, how many shillings in 18 guineas? Ans. 504.

32. A gentleman wishes to draw off 5 hhd. of wine into bottles of 1 pt. each; how many bottles will be required? Ans. 2520.

33. A farmer gives his sheep one bushel and one peck of corn a day, giving to each sheep half a pint; how many sheep has he?
Ans. 160.

34. How many cubick blocks, each containing one solid inch, can be made from a stick of timber containing 39 solid ft. 500 in., allowing 230 solid in. for waste? Ans. 67662.

When a number is to be changed to a higher denomination,

RULE.

Divide the given number by as many as it takes of that denomination to equal one in the denomination required.

35. In 5792 oz. Avoirdupois, how many pounds?

Operation.

```
16)5792(362 Ans.
   48
   ——
    99
    96
    ——
     32
     32
     ——
```

Illustration.—The divisor being equal to one in the next greater denomination, every time we subtract it from the dividend, we take units enough from the dividend, to make one in the next higher denomination; therefore as many times as we can subtract the divisor from the dividend, we shall have so many units of the higher denomination in the quotient, for we place one in the quotient every time we subtract the divisor from the dividend.

The quotient will equal the dividend if there be no remainder, for every time we take a number from the dividend equal to the divisor, we put 1 in the quotient which is just equal to that number. If there be a remainder, the quotient and remainder will equal the dividend.

In the last question, 16 oz. being equal to 1 pound, by taking that number from the given ounces we subtract 1 pound, and as many times as we can subtract 16 from the 5792 ounces, so many pounds there will be;—but the quotient expresses this number of times, therefore the quotient is pounds, and represents the same quantity which the dividend did.

36. In 24000 qr: how many pence?
Ans. 6000.

37. In 6000 d. how many shillings?
Ans. 500.

38. In 500 s. how many pounds?
Ans. 25.

39. How many ounces in 2400 pwt.?
Ans. 120.

40. How many pounds Troy in 120 oz.?
Ans. 10.

41. In 31360 lb. how many quarters?
Ans. 1120.

42. In 1120 qr. how many hundred?
Ans. 280.

43. How many tons in 280 cwt.?
Ans. 14.

44. How many rods in 2455200 ft.?
Ans. 148800.

45. How many furlongs in 148800 r.?
Ans. 3720.

46. In 3720 fur. how many miles?
Ans. 465.

Note 2.—From several of the last questions, it may be seen that we have only to divide the several quotients as directed by the last rule, when a denomination is to be changed to another greater than that which is next to the given one.

If there be remainders, they will be of the same denomination as the dividend from which they were obtained, and they must be brought to the right hand of the last quotient.

47. In 1994435 s. how many days?

Operation.

```
6 | 0 ) 1 9 9 4 4 3 | 5
        ---------------
     6 | 0 ) 3 3 2 4 | 0 minutes and 35 s. remain.
             -------
               5 5 4   hours.
         2 4 ) 5 5 4 (23 d. 2 h. 0 m. 35 s.   Ans.
               4 8
               ---
                 7 4
                 7 2
                 ---
                   2
```

48. In 36567890 square inches, how many acres?
Ans. 5 A. 3 R. 12 r. 206 ft. 98 in.

For dividing by 272¼ in the last question, see page 80.

49. 160 square rods equal 1 A. How many acres in 960098 square rods? Ans. 6000 A. 98 r.

50. In 960098 solid ft. of wood, how many cords?
Ans. 7500 C. 98 ft.

51. How many tons of square timber in 46073 ft.?
Ans. 921 T. 23 ft.

52. In 136097 d., how many dollars?
Ans. 189 dol. 1 s. 5 d.

53. In 56328 d. how many eagles,—an eagle being equal to 10 dollars?
Ans. 78 eagles, 2 dol. 2 s.

54. How many chaldron of corn in 463049 pt.?
Ans. 200 ch. 35 bus. 4 qt. 1 pt.

Questions to be worked by Reduction and Subtraction.

1. How much more land in a field which contains 179087 rods, than in one measuring 987 A. 3 R.?
Ans. 131 A. 2 R. 7 r.

2. A man wishes to draw the wine from a hhd. into

pint bottles; having but 499 bottles there will be some remaining in the hhd.—how much?

Ans. 2 qt. 1 pt.

3. A man lends his neighbour a quarter of veal weighing 19 lb. 11 oz. 11 dr. for which the borrower returns another, weighing 316 oz. Does the borrower pay or not? Ans. He overpays 5 dr.

PROMISCUOUS QUESTIONS.

The following questions not belonging to any particular rule, are inserted here by themselves, and their various operations explained.

1. A man has 101 A. 3 R. 32 r. of land, which he wishes to fence into lots of 12 A. 2 R. 39 r. each. How many lots will there be?

Operation.

```
 A.      R.      r.         A.       R.      r.
 1 2     2       3 9      1 0 1      3       3 2
   4                          4
 ----                     -----
 5 0                      4 0 7
 4 0                        4 0
------                   -------
2 0 3 9       2 0 3 9)1 6 3 1 2(8  Ans.
                      1 6 3 1 2
                     ----------
```

Explanation.—As many times as we can take 12 A. 2 R. 39 r., (what each lot is to contain,) from the 101 A. 3 R. 32 r., there will be so many lots. As it is not generally so easy to divide one compound number by another, as it is to reduce them to the lowest denomination mentioned in either, and then divide, we reduce the divisor and dividend to rods. By illustration, page 121, 16312 rods equal 101 A. 3 R. 32 r., and 2039 rods equal 12 A. 2 R. 39 r.; consequently we can take 2039 rods from 16312 rods as many times as we can 12 A. 2 R. 39 r. from 101 A. 3 R. 32 r.

The same method may be observed in all similar questions.

2. If 1111 gal. of brandy be put into casks containing 47 gal. 3 qt. 1 pt. each, how many casks will be required?

Ans. 23 casks, and 9 gal. 3 qt. 1 pt. over.

3. How many times will a wheel, which is 15 ft. 7 in. in circumference, turn over in the distance of 2 M. 4 fur. 38 r. 11 ft.? Ans. 888.

Note 1. If the multiplier contain a fraction, and the upper part of the fraction be 1, divide the multiplicand by the under part of the fraction, and add the quotient to the product of the other figures.

4. In 19 £. how many crowns, a crown being 6 s. 7 d.? Ans. 57, and 4 s. 9 d. over.

5. In 12 £. 18 s. how many dollars? Ans. 43.

6. How many parcels each 7 lb., can be taken from 15 cwt. 2 qr. 18 lb. of sugar, neat weight?

Ans. 224.

7. How many pairs of sugar tongs, each weighing 7 pwt. 1 gr., can be made from 3 lb. of silver, if 8 gr. are allowed for waste?

Ans. 102, and 1 pwt. 10 gr. over.

8. If a man step 2 ft. 10 in., how many steps will he take in travelling 4 fur. 11 r. 11 ft. 10 in.?

Ans. 1000.

9. How many guineas 28 s. each, in 100 £.?

Ans. 71, and 12 s. over.

10. In 657012 dollars, how many pounds, how many guineas, and how many crowns?

Ans. £ 197103, 12 s.
Guineas 140788, 8 s.
Crowns 598795, 4 s. 11 d.

11. How far apart must 12 spokes be inserted in a hub, whose circumference is 3 ft. 6 in. if each spoke be 1½ in. thick? Ans. 2 in.

12. How many trees can be set on a right line, 16 r. 6 ft. in length, if the trees stand 15 ft. asunder?

Ans. 19.

Note 2. The number of trees will be one more than the number of spaces.

13. A gentleman wishes to set 20 trees at equal distances on a line 18 r. 7 ft. in length; how far apart must the trees be? Ans. 16.

14. In 14 E. E. how many yards? Ans. 17 and 2 qr.

15. In 14 E. Fr. how many yards? Ans. 21.

16. In 14 E. Fl. how many yards? Ans. 10 and 2 qr.

17. Divide 735 dollars between A and B, giving to A 2 shares and to B 3.

```
2 number of A's shares,          5 ) 7 3 5
3 number of B's shares,              -----
—                                 $ 1 4 7 the value of
5 sum of the shares.                        1 share.

        1 4 7                         1 4 7
            2                             3
        -----                         -----
      $2 9 4 A's part.              $4 4 1 B's part.

        Proof.
A's part 2 9 4
B's part 4 4 1
         -----
         7 3 5
```

Explanation.—Had A and B owned the money equally between them, we should have divided by 2, to have found the part of each, but as one owns a greater part than the other, we must add the shares of the whole together, and divide by the sum, the quotient will be the value of 1 share; and multiplying the value of one share by the number of shares which one person owns must give his part.

18. Three men, A, B and C, buy a ticket in company, for which they pay $10. A pays 1 dol., B 3 dol. and C the remainder. If the ticket draw $12000, what will be the share of each?

Ans. A, $1200, B, $3600 and C, $7200.

19. A retailer draws off a pipe of brandy into casks containing 10 gal. 1 pt.; how many casks does it fill?

Ans. 12, and 4 gal. 2 qt. over.

20. If a tun of wine be drawn into quart and pint

bottles, of each kind an equal number, how many of each kind will be required? Ans. 672.

Explanation.—Were we to draw the wine in the last question, into a measure containing 3 pints, each measure full would fill a bottle of each kind; consequently, as many times as we can take 3 from the number of pints contained in a tun, there will be so many bottles of each kind.

21. Three men A, B, and C perform a journey in company, the expense of which was 91 dols. B is to pay 3 times as much as A, and C three times as much as B; what does each man pay?

Ans. A pays $7, B $21, and C $63.

* * *

SECTION VII.

FRACTIONS.

DEFINITIONS.

A Fraction is a part of an unit, or a part of one single thing of any kind, expressed by two numbers, one written above the other, with a line between them; thus, $\frac{6}{11}$ is a fraction expressed by the numbers 6 and 11.

A Denominator is the under number of a fraction; 11 is the denominator in the fraction $\frac{6}{11}$.

A Numerator is the upper number of a fraction; 6 is the numerator in the fraction $\frac{6}{11}$.

A Simple Fraction is a fraction having but one numerator and one denominator; as, $\frac{1}{5}$.

A Compound Fraction has more than one numerator and denominator, with the word *of* written between its parts; as, $\frac{1}{2}$ of $\frac{1}{4}$. The first part of a compound fraction is a fraction of that part which follows it; thus, in the fraction $\frac{1}{2}$ of $\frac{1}{4}$, the $\frac{1}{2}$ is a fraction of the $\frac{1}{4}$, and the $\frac{1}{4}$ is a fraction of an unit.

A Whole Number is the name given to a simple number to distinguish it from a fraction, when they are written in connection. In the expression $15\frac{2}{7}$, 15 is the whole number and $\frac{2}{7}$ the fraction.

A Mixed Number is a whole number and a fraction; as, $3\frac{2}{3}$.

An Improper Fraction has its numerator equal to, or greater, than its denominator; as, $\frac{3}{3}$, or $\frac{5}{2}$.

A Proper Fraction has its numerator less than its denominator; as, $\frac{6}{7}$, or $\frac{1}{9}$.

The Terms of a Fraction is a word denoting both the numerator and denominator of the same fraction, when both are to be expressed by one word.

◆

EXPLANATION OF FRACTIONS.

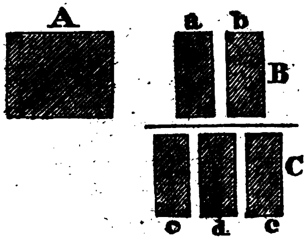
It is frequently necessary to express parts of a quantity less than that part denoted by an unit. If we wish to represent by figures one barrel of flour and a part of another barrel, the figure 1 will represent the whole barrel, and the part of the other must be represented in a different manner. Suppose we have one barrel and three-quarters of another;—to show into how many parts that barrel is divided, of which we have only a part, we must make use of one figure, expressing the number of parts into which it is divided: and this number will be the denominator of a fraction. We must also make use of a figure denoting the number we have of those parts into which the barrel is divided; and this figure will be the numerator of the fraction. The one barrel and three-quarters of flour would be expressed thus, $1\frac{3}{4}$; the figure 1 at the left hand represents the whole barrel, and the figure 4 denotes how many parts another barrel is divided into, and is the denominator; the figure 3 shows how many of those parts we have, and is the numerator, The denominator is so called because it denotes how many parts one, or an unit, is divided into.

The numerator is so called because it expresses, or numbers the parts we possess.

Again, let it be required to express one yard of cloth, and two-thirds of another yard. To exhibit the fraction in a clearer light, we shall make use of the following Plate:

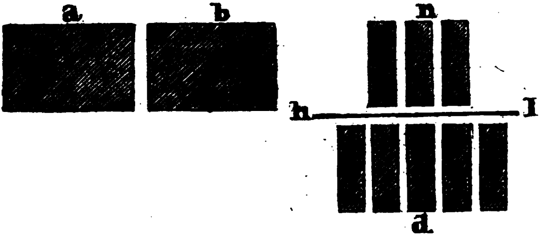
PLATE 1.

1 and $\frac{2}{3}$ yards of cloth.



In plate 1, let that part of the plate under A represent one yard of cloth, the figure 1, standing above it, signifies the same thing. Let the part of the plate C represent another yard of cloth, equal to that represented by A; but let us suppose it divided into three equal parts, that is, into thirds. The parts above *c*, *d*, and *e*, will each represent one-third of a yard, and the three pieces a whole yard. We do not possess the whole yard denoted by C, but suppose a whole yard divided into the three equal parts, *c*, *d*, *e*, and this supposed yard is used as a denominator to show what each part in the numerator is equal to. The figure 3, the denominator of the fraction $\frac{2}{3}$ signifies the same three parts as the letters *c*, *d*, and *e*, that is, the supposed yard. Let the part of the plate B denote two pieces of cloth, each piece just as large as the piece *c*, or *d*, in the part C. Then the part *a* will represent one-third part of a yard, and the parts *a* and *b* will represent $\frac{2}{3}$; and these two parts are denoted by 2, the numerator of the fraction $\frac{2}{3}$. We supposed in the first place, that we had one yard, and two-thirds of another. The part of the plate A denotes the whole yard, and the part C represents a supposed yard, of which we have only a part. As we have two-thirds of another yard, we may suppose the third *c* and the third *d*, removed, and placed above the horizontal line, in the places *a*, *b*; then *e* would show what part is remaining to make up another full yard. But as *a* and *b* are just equal to *c* and *d*, we may as well let *c* and *d* remain, and *a* and *b* will show what part of another yard we have.

PLATE 2.



In plate 2, let *a* and *b* denote two single things of any kind, or two units; and let *d* represent a supposed thing or unit, just as large as *a*. If *d* be divided into 5 equal parts, one of the parts will be a fifth part of the whole unit denoted by *d*, and the 5 parts will be equal to *a* or *b*, that is, equal to an unit. The part *n* is divided into three equal divisions, each equal to one of the divisions in *d*, therefore one of the divisions in *n* is one fifth part of an unit, and the three divisions in *n* are three-fifths of an unit. *d* and *n* represent a fraction, *d* the denominator, and *n* the numerator. The whole would be thus expressed by figures, $2\frac{3}{5}$. The figure 2 represents *a* and *b*, 5 the same as *d*, and 3 the same as *n*. If any particular quantity were denoted by the several parts of the plate, we should possess the parts represented by *a*, *b*, and *n*, all the parts above the line *h I*.

DERIVATION OF FRACTIONS.

A fraction is derived from division. *If 15 dollars be equally divided between 4 men, how many dollars will each man receive?*

4) 1 5 ($3\frac{3}{4}$ Each man will receive $3\frac{3}{4}$ dollars.
    1 2
    ——
      3

By taking 4 dollars from the whole number, we take away dollars enough to give each man one; and by putting 1 in the quotient every time we take away 4 from the whole number of dollars, the quotient will show how many dollars each man will receive. By working the question, we find that 4 can be taken 3 times from the 15, and 3 remains. In 12 dollars, the number we have already subtracted, there are enough to give each man 3. As the number of dollars left is not equal to the number of men, there will not be another dollar for each man, but each will receive a part of another dollar. If we suppose the 3 dollars divided into quarters, the number would be 12, and each man would receive 3 quarters from the 12 quarters, in the same manner that he received 3 dollars from the 12 dollars. 4, the divisor, shows how many units of the dividend it requires that we may place an unit in the quotient; therefore, by making the divisor a denominator to a fraction, the 4 will show the number of units required to be taken from the dividend, to express one unit in the quotient. The numerator 3, which is the remainder, shows how many units we have left from the dividend, towards making another unit in the quotient.

Again, let it be required to divide 19 by 12.

1 2) 1 9 ($1\frac{7}{12}$
        1 2
        ——
          7

In this example, it takes 12 in the dividend to make 1 in the quotient. We may suppose 1, the whole number in the quotient, to be so much larger than the units in the dividend, that it requires 12 units in the dividend to make one unit as large as that in the quotient. By setting 12 as a denominator, it shows how many of the small units in the dividend are required to make one unit in the quotient. The units contained in 7 are just as large as those contained in 19, the dividend, and are considered just as large as those in 12, the divisor; therefore, if we place 7 for a numerator, it will show how many small units we have towards making another large unit in the quotient. But if it take 12 units in the dividend to make 1 in the quotient, an unit in the dividend is one-twelfth part of an unit in the quotient, and the 7 must be 7 twelfths of an unit in the quotient.

Let the divisor be any number whatever, it will always require as many units in the dividend to equal one in the quotient, as there are ones in the divisor; therefore, the divisor will always be the proper number to show how many small units like those in the remainder, it will require to make an unit in the quotient; and the numerator being the remainder, will always denote the number of those small units we have.

It may here be observed, that the units in the quotient are not always larger than those in the dividend, but as all operations in division are performed in the same manner as if they were, it is proper to consider them as such, in explaining the subject; and the scholar will understand them much better in this way, than in any other.

As we make use of the same figures in fractions as in whole numbers, it is difficult, many times, to explain the different operations in them, without using the word unit. For the sake of distinction, therefore, we shall call the parts of an unit denoted in a fraction, *fractional units.*

It may not at first appear to the learner, how a fraction, standing by itself, arises from division. If it were required to divide one dollar equally between 3 men, the one dollar becomes the dividend, and the

3 men the divisor. The divisor being larger than the dividend, we cannot actually divide, but the quotient is properly expressed by writing the divisor under the dividend, thus, $\frac{1}{3}$. Had the dividend been as large as the divisor, the quotient would have been 1, but as it is less, it is the same as a remainder, and by setting the divisor under the 1, the fraction shows what part of a dollar each man would receive.

From what has been said we obtain the following,

GENERAL PRINCIPLES OF FRACTIONS,

1. The denominator to a simple fraction is always equal to an unit in the whole number to which the fraction belongs.
2. The denominator and numerator are always of the same denomination; that is, a fractional unit in the denominator is always equal to a fractional unit in the numerator.
3. The value of a fraction is determined by dividing the numerator by the denominator.

METHOD OF READING FRACTIONS.

The numerator and denominator are read in the same manner as simple numbers, the numerator first. $\frac{127}{354}$ is read, one hundred and twenty-seven, three hundred and fifty-fourths; and $\frac{25}{326}$ is read twenty-five, three hundred and twenty-sixths.

VULGAR FRACTIONS.

DEFINITIONS.

A Prime Number is a number which cannot be divided by any number without a remainder, except by itself or an unit, as 5, 11, 13, &c.

A Multiple is a number which can be divided by some other number without a remainder; and is called a multiple, because it can be produced by multiplying two other numbers together. 9 is a multiple of 3.

A Common Multiple of two or more numbers, is any number which can be divided by each of those numbers without a remainder. If it be the least number that can be divided by the numbers, it is called the *least common multiple;* thus, 54 is a common multiple of 6, 9 and 18; but 18 is their least common multiple.

A Common Divisor, or Common Measure, is a number that will divide two or more numbers without a remainder. If it be the greatest number that will divide without a remainder, it is called the greatest common divisor, or measure.

PROBLEM I.

To find the least common multiple of several numbers.

RULE.

1. Divide the given numbers by any number that will divide two or more of them without a remainder, and set the quotients and the numbers which have not been divided in a line below.

2. Divide the numbers brought down as before, and so continue to do until there are no two numbers that can be divided by one number without a remainder. Multiply all the divisors, the last quotients, and the last undivided numbers together, the last product will be the least common multiple of the given numbers.

1. What is the least number that can be divided by 8, 12 or 5, without a remainder?

Operation.

```
    4 ) 8    1 2    5
    2  -------------
   --   2     3     5
    8
    3
   ---
  2 4
    5
 -----
1 2 0  Ans.
```

Illustration.—If one number can be divided by another without a remainder, and that number be multiplied by any simple number, the product can be divided by the same divisor without a remainder. 6 can be divided by three without a remainder. If 6 be multiplied by 4, the product 24 can be divided by 3 without a remainder, for 6 is made up of two 3's, and every time we put 6 into the product, we put in a certain number of 3's. The product of a divisor multiplied by a quotient, can always be divided by the dividend, for the product will equal the dividend. 8, one of the given numbers in the last question, will divide the first product 8. 3 is the quotient of 12 divided by 4, and 8 the first product is two 4's. Now if we multiply 8 by 3, the product is two 12's. 24, the second product, is made up of acertain number of 8's, or 12's ; 120 is made up of a certain number of 24's or 5's. But as 24 is made up of a certain number of 8's, or 12's, 120 must be ; therefore 120 the continued product of the divisor, quotients, and undivided number, can be divided by 8, 12, or 5, without a remainder.

120 is the least number that can be divided by the three given numbers without a remainder, for if we divide one of the under numbers by itself, we must multiply by that divisor, which would produce the same product ; and if we could divide one of the under numbers by some other number, multiplying by

the new divisor and quotient, would increase the product as much as the number divided. The same might be shown as it respects any other numbers, for the last product would always be made up of each of the given numbers a certain number of times.

2. What is the least common multiple of 20, 6, 8, 15, 7?

$$\begin{array}{r|rrrrr} 5) & 20 & 6 & 8 & 15 & 7 \\ \hline 4) & 4 & 6 & 8 & 3 & 7 \\ \hline 3) & 1 & 6 & 2 & 3 & 7 \\ \hline 2) & 1 & 2 & 2 & 1 & 7 \\ \hline & 1 & 1 & 1 & 1 & 7 \end{array}$$

$5 \times 4 = 20 \times 3 = 60 \times 2 = 120 \times 7 = 840$ Ans.

3. What is the least common multiple of 2, 3, 7, 9?
Ans. 126.

4. What is the least number that can be divided by 7, 11, or 13? Ans. 1001.

5. What is the least common multiple of 26, 100, 234, 1050? Ans. 81900.

PROBLEM II.

To find the greatest common divisor, or measure of two numbers.

RULE.

Divide the greater number by the less, and the divisor by the remainder. Continue to divide the last divisor by the last remainder until nothing remains; the last divisor will be the greatest common divisor, or measure.

1. What is the greatest common divisor, or measure of 125 and 2225?

Operation.

```
      125)2225(17
          125
          ---
           975
           875
           -------
           100)125(1
               100
               ---
                25)100(4
                   100
                   ---
1250      2225
 875       125
 100      ----
 100      2350 sum of the given numbers.
  25
----
2350 sum of the subtrahends and last divisor,
         or remainder.
```

Illustration.—In the operation of the last question, 25 the last divisor or remainder, will divide itself; it also divides the last dividend without a remainder. But the last divisor and the last dividend added together make 125, the least given number, which is 5 times 25, consequently 25 will divide it without a remainder. All the subtrahends are made by multiplying 25, or numbers made up of 25's, therefore all the subtrahends are divisible by 25. It will be seen by the operation, that the sum of the subtrahends and last divisor, equals the sum of the given numbers; the given numbers therefore are composed of 25's, and can be divided by 25 without remainders. By examining the operations of other questions, it will be seen that all the subtrahends will be composed of the last divisor, or some multiple of the last divisor, a certain number of times. The sum of the subtrahends and last divisor will always equal the given numbers, for making the divisor the next dividend, we always subtract from some part of the given numbers, therefore the last divisor will always divide the given numbers without a remainder.

2. What is the greatest common divisor of 24 and 160? Ans. 8.

3. Required the greatest common measure of $\frac{1836}{3267}$. Ans. 27.

4. What is the greatest common measure of $\frac{26}{16678}$? Ans. 2.

5. Find the greatest number that will divide both parts of the fraction $\frac{101}{169606}$ without a remainder. Ans. 1.

6. If I have two quantities of coal, one 468 bushels, the other 612, how many bushels must the greatest measure contain that will exactly measure both quantities? Ans. 36.

REDUCTION OF FRACTIONS.

Reduction of fractions is changing the form of fractions without altering their values.

The same part or parts of an unit may be expressed by different fractions. We can only tell what part of an unit a fraction is, by comparing the numerator with the denominator. The fractions $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, all express the same part of an unit, for in each the numerator is half the denominator, and each denominator being equal to an unit, the value of each fraction is $\frac{1}{2}$ of an unit. By dividing both terms of the three last fractions by 2, 3, and 4, respectively, we shall obtain $\frac{1}{2}$ for the quotient of each fraction. When both terms of the fraction are divided by the same number, the quotients will be a fraction expressing the same part or parts of an unit as the numbers divided, for they are both lessened by the same number.

If we divide both parts of $\frac{2}{4}$ by 2, 2) $\frac{2}{4}$ ($\frac{1}{2}$, we take half the numerator for a new numerator, and half the denominator for a new denominator. One half of a number must be the same part of a half of another number, that one whole number is of the other.

2 is half of 4, and 1 is half of 2, so that 1 is the same part of 2, that 2 is of 4, that is, $\frac{1}{2}$, is the same part of a unit as $\frac{2}{4}$. If we divide both parts of a fraction by any other number, the quotients will be a fraction expressing the same value as the given fraction, for dividing by the same number, we take the same part of the numerator that we do of the denominator; thus, $6\,)\,\frac{12}{18}=\frac{2}{3}$;—here we take a sixth part of the numerator, and a sixth part of the denominator, and find 2 to be the same part of 3, that 12 is of 18.

Note 1.—When no number greater than 1 will divide both terms of the fraction without a remainder, the fraction is said to be in its *lowest terms.*

From the above reasoning we obtain the truth of the following

RULE,

For reducing a fraction to its lowest terms.

Find the greatest common measure, and divide both terms of the fraction by it, the quotients will be a fraction expressing the same value as the given fraction.

Note 2.—When the terms of the fraction are small numbers, we can generally discover at sight what number will divide both without a remainder, if the fraction be not then in its lowest terms, we must continue dividing in a similar manner until it is.*

1. Reduce $\frac{235}{1175}$ to its lowest terms.

Operation.

To find the greatest common measure.	Given numerator.
235)1175(5	235)235(1 new numerator.
1175	235

---

* If the two right hand figures of the fraction be even numbers, the terms of the fraction are divisible by 2; if 5's, divisible by 5; if ciphers, divisible by 10, or by 5. If the sum of the figures in each term be divisible by 3 or 9, both terms are divisible by 3 or by 9.

```
           Given denominator.
235 ) 1175 ( 5 new denominator.
      1175
      ----
```

The fraction in its lowest terms is $\frac{1}{5}$.

2. Reduce $\frac{120}{280}$ to its lowest terms.

Operation as directed in the Note.

$\frac{120}{280} \div 4 = \frac{30}{70} \div 5 = \frac{6}{14} \div 2 = \frac{3}{7}$ the answer.

3. Reduce $\frac{1378}{1484}$ to its lowest terms. Ans. $\frac{13}{14}$.

4. Reduce $\frac{3402}{15309}$ to its lowest terms. Ans. $\frac{2}{9}$.

5. What fraction in its lowest terms is equal to $\frac{10017}{285489}$? Ans. $\frac{1113}{31721}$.

6. In $\frac{799}{2397}$, what part of the denominator is the numerator? Ans. $\frac{1}{3}$.

To change a mixed number to the form of a fraction, that is, to an improper fraction.

RULE.

Multiply the whole number by the denominator, and to the product add the numerator; the sum placed over the denominator will form an improper fraction, equal in value to the mixed number.

1. What improper fraction is equal to $10\frac{1}{5}$?

$10 \times 5 + 1 = \frac{51}{5}$ the answer.

Illustration.—The denominator being equal to an unit in the whole number, (see explanation of fractions, page 130,) if it be placed in the numerator as many times as there are units in the whole number, the numerator will contain as many units as the whole number, and by adding the given numerator, the sum will equal the whole number and the given fraction. In multiplying the whole number by the denominator, we place the denominator in the product once for each unit in the whole number, or multiplicand, (see illustrations, pages 36 and 43,) or in other words, we put into the product the number of parts the unit is supposed divided into, as many times as there are units in the whole number, and this product we make a part of the numerator by adding it to the given numerator. By keeping the multiplier for a denominator, it shows how many fractional units in the numerator are required to make a whole unit. If an unit be divided into 5ths, there would be 5 fifths. If we multiply a whole number by 5, we place 5 in the product for each unit in the whole number, and by so doing, the product expresses the value of the whole number divided into 5ths.

There are 50 fifths in 10, as may be seen in the work of the last question, and by adding the 1 fifth given, the whole value of the mixed number, $10\frac{1}{5}$, is 51 fifths, or $\frac{51}{5}$.

2. What improper fraction is equal to $37\frac{3}{7}$? Ans. $\frac{262}{7}$.
3. Change $123\frac{1}{14}$ to the form of a fraction. Ans. $\frac{1733}{14}$.
4. Change $1702\frac{1}{125}$ to an improper fraction. Ans. $\frac{212751}{125}$.
5. How many 5ths in 76? Ans. 380.
6. How many 8ths in 139? Ans. 1112.
7. How many 49ths in 3? Ans. 147.
8. How many shares in that library, which is worth 720 dollars, if each share be worth $\frac{1}{3}$ of a dollar? Ans. 2160.
9. If 8 dollars are to be distributed among a certain number of scholars, as premium money, and each scholar is to receive $\frac{1}{7}$ of a dollar, how many scholars will receive a premium? Ans. 56.
10. How many testaments, at $\frac{1}{5}$ of a dollar a piece may be bought for 31 dollars. Ans. 155.

To change an improper fraction to a mixed number.

RULE.

Divide the numerator by the denominator; the quotient will be the whole number, and the remainder set over the denominator, will form the fraction.

1. Change $\frac{172}{8}$ to a mixed number.

$$\begin{array}{r} 8\,)\,1\ 7\ 2 \\ \hline 2\ 1\frac{4}{8} \end{array} = 21\tfrac{1}{2}.$$ the answer.

Illustration.—In the last example, the 172 are 8ths, and it requires 8 eighths to make one unit; therefore, as many times as we can take 8 from the 172, there will be so many units. 4, the remainder, being 8ths, and half as many as it requires to make an unit, will constitute $\frac{1}{2}$ of another unit.

The rule is true as it respects any other improper fraction, for the denominator will always show how many fractional units must be taken from the numerator, to equal a whole unit, or an unit in a whole number.

From this illustration and the preceding one, the propriety of dividing the numerator of a fraction by the denominator, to find the value of the fraction, is manifest.

2. Change $\frac{16907}{73}$ to a mixed number.
Ans. $231\frac{44}{73}$.

3. What number is equal to $\frac{9720}{24}$?
Ans. 405.

4. How many units in 87030 sixths?
Ans. 14505.

5. What mixed number is equal to $\frac{32059406}{127047}$?
Ans. $252\frac{43562}{127047}$.

6. How many dollars will 135 yards of riband come to at $\frac{1}{9}$ of a dollar per yard? Ans. 15.

7. If a man drink a gallon of cider in 4 days, how many days will he be drinking 78 quarts?
Ans. 78.

8. If a man travel a mile in $\frac{1}{5}$ of an hour, how many hours will he be travelling 113 miles?

Ans. $22\frac{3}{5}$.

To reduce a compound fraction to a simple one.

RULE.

Multiply all the numerators together for a new numerator, and all the denominators together for a new denominator; the new numerator and denominator will be a fraction expressing the same value as the compound one.

Note 1.—If any part of a compound fraction be a mixed number, it must be changed to an improper fraction, and the improper fraction used instead of the mixed number. If a whole number be connected with the fraction, it must be changed to the form of a fraction by placing 1 for a denominator. The value of the whole number will not be altered, for dividing the numerator by one, the quotient will be the same as the given number.

1. What is $\frac{1}{2}$ of $\frac{1}{4}$ of a dollar?

$\frac{1}{2}\times\frac{1}{4}=\frac{1}{8}$ of a dollar, the answer.

Illustration.—A part of a thing is less than the whole; a part of *that* part is still less. If we multiply 1 by 1 the product is 1. $\frac{2}{2}$ equal 1, and $\frac{4}{4}$ equal 1; if we multiply $\frac{2}{2}$ by $\frac{4}{4}$, the product is $\frac{8}{8}$, or 1, for dividing the numerator by the denominator, the quotient is 1, which is equal to the multiplicand. If we multiply by less than 1, that is, by a proper fraction, the product must be less than the multiplicand, for we can take only such a part of the multiplicand as the multiplier is of an unit. Once $\frac{1}{2}$ is $\frac{1}{2}$; but let us multiply $\frac{1}{2}$ by $\frac{1}{2}$, the product must be less than when we multiplied $\frac{1}{2}$ by 1; it can be only half as large, $\frac{1}{4}$; and $\frac{1}{4}$ we obtain by multiplying one numerator by the other, and one denominator by the other denominator. In the first question, $\frac{1}{2}$ of $\frac{1}{4}$ of a dollar is required. Let the 4 quarters of a dollar be each divided into two equal parts, the whole dollar will be divided into 8 equal parts; but when we divide $\frac{1}{4}$ into two equal parts, one of these parts is half the quarter; it is also $\frac{1}{8}$ of the whole dollar. If both of the numerators be 1's, the numerator of the product will be 1, but multiplying the denominator of $\frac{1}{4}$ by 2, the other denominator, the denominator in the answer $\frac{1}{8}$, is twice as large as in $\frac{1}{4}$, and the part of the dollar expressed by the numerator of $\frac{1}{8}$, is only half as large as the part expressed by the numerator of $\frac{1}{4}$, for multiplying by 2, the denominator becomes twice as large. Or in other words, when we say $\frac{1}{8}$ of a dollar, the dollar is supposed to be divided into 8 equal parts, and we have one of them; but when we say $\frac{1}{4}$ of a dollar, the dollar is supposed divided into half as many parts, consequently one of these parts must be twice as large.

A fraction properly expresses a quotient. $\frac{10}{5}$ signifies a quotient of 2, for $10 \div 5 = 2$. But the quotient denoted by a proper fraction cannot be expressed by a single number. If we were required to divide 1 dollar equally between 3 men, we should be required to divide 1 by 3, and $\frac{1}{3}$ would denote the quotient, for we cannot actually divide the dividend 1, which is the numerator, by the divisor 3, which is the denominator. Let it be required to find what part of a shilling $\frac{1}{3}$ of $\frac{3}{4}$ is. $\frac{3}{4}$ implies that 3 is to be divided by 4. Now if we multiply $\frac{3}{4}$ by $\frac{1}{3}$ as directed in the rule, we obtain $\frac{3}{12}$, the same numerator, as in the last part of the fraction, but a denominator 3 times as large. It is easy to perceive that the quotient 3 divided by 12, can be only a third part as large as the quotient of 3 divided by 4;— $\frac{3}{4}$ of a shilling is 9 pence, and $\frac{1}{3}$ of 9 pence is 3 pence, or $\frac{3}{12}$ of a shilling, which is equal to $\frac{1}{3}$ of $\frac{3}{4}$.

What is $\frac{2}{3}$ of $\frac{3}{4}$ of a shilling? The answer must be twice as large as the answer to $\frac{1}{3}$ of $\frac{3}{4}$ of a shilling. By working this example, we obtain the same denominator, or divisor, as before; but multiplying the numerator of $\frac{3}{4}$ by 2 the numerator, or dividend, is twice as large, for $\frac{6}{12}$ of a shilling implies twice as many pence as $\frac{3}{12}$.

The truth of the rule rests on this general principle, that the greater the denominators are when compared with the numerators, the less part of an unit, or of another fraction, they express; and if the denominators be larger than the numerators, their product must in the same degree be larger than that of the numerators, consequently the answer must be a less part of an unit than the last part of the compound fraction. But if the numerators are the largest, each part of the compound fraction denotes more than once the next part, or an unit, if it be the last part, and their product will be in the same degree larger than that of the denominators.

2. What is $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$ of a dollar?
Here $\frac{1}{2}$ of $\frac{1}{3}$ is $\frac{1}{6}$, and $\frac{1}{6}$ of $\frac{1}{4}$ is $\frac{1}{24}$ the answer.

3. What is $\frac{1}{5}$ of $\frac{6}{7}$ of $\frac{2}{17}$? Ans. $\frac{12}{595}$.

4. What part of an unit is $\frac{1}{9}$ of $\frac{1}{3}$? Ans. $\frac{1}{27}$.

5. If $\frac{1}{4}$ of a bushel of pears be equally divided between 3 boys, what part of a whole bushel will each boy have?

Explanation.—Each boy is to receive $\frac{1}{3}$ of the pears, which is $\frac{1}{3}$ of $\frac{1}{4}$ of a bushel, and $\frac{1}{3}$ of $\frac{1}{4}$ is $\frac{1}{12}$ of a bushel.

6. If I have $\frac{1}{2}$ bushel of wheat, and give $\frac{2}{5}$ of it to a poor neighbour, what part of a bushel do I give him? Ans. $\frac{2}{10}$, or $\frac{1}{5}$.

7. A man owning $\frac{3}{11}$ of a share in a certain canal stock, sold $\frac{2}{17}$ of his part; what part of a share did he sell? Ans. $\frac{6}{187}$.

8. If 18 men share equally $\frac{2}{9}$ of a captured ship, what part of the ship does each man possess? Ans. $\frac{2}{162}$, or $\frac{1}{81}$.

Explanation.—One man's part is $\frac{1}{18}$ of $\frac{2}{9}$.

9. If I, with 25 others, build $\frac{3}{4}$ of a mile of road, and each build an equal proportion, what part of a mile do I build? Ans. $\frac{3}{104}$.

10. At $\frac{3}{7}$ of a dollar per yard, what part of a dollar will $\frac{1}{5}$ of a yard cost? Ans. $\frac{3}{35}$.

11. If a man have coffee worth 25 cents a pound, what part of a dollar must he receive for 11 ounces?

Explanation.—25 cents$=\frac{1}{4}$ of a dollar, and 11 ounces$=\frac{11}{16}$ of a pound; then the question is, what part of a dollar is $\frac{11}{16}$ of $\frac{1}{4}$? Ans. $\frac{11}{64}$.

12. What number is equal to $\frac{3}{11}$ of $\frac{22}{3}$?

Ans. 2.

Note 2.—When a part of a whole number is required, and the numerator of the fraction expressing that part, is one, we shall obtain the part demanded by dividing the whole number by the denominator.

13. If a man own $\frac{1}{11}$ of a lot of steel worth 1000 dollars, what is his part worth? Ans. \$$90\frac{10}{11}$.

14. If a man's income be 725 dollars a year, what is it per month?

Explanation.—One calendar month is $\frac{1}{12}$ part of a year, and the answer is $\frac{1}{12}$ of $725=$\$$60\frac{5}{12}$.

15. If 645 dollars be divided into 15 equal parts, what will one of the parts be; that is, what is the 15th part of 645? Ans. 43.

16. What is $\frac{2}{15}$ of 645?

Explanation.—In the last question we found $\frac{1}{15}$ of 645 to be 43, and $\frac{2}{15}$ must be twice as much, therefore if we divide the whole number by the denominator, and multiply the quotient by the numerator, we shall have a correct answer. But if we multiply the whole number by the numerator first, and then divide by the denominator, we shall obtain the same answer, since the dividend will be twice as large. The latter method is generally the most convenient, since we shall not so often have a fraction in the number to be multiplied. In dividing by the denominator, we take such a part of the whole number as would be expressed if the numerator were 1, and multiplying by the numerator will give that part as many times as there are parts expressed by the numerator.

17. What number is equal to $\frac{3}{8}$ of 24?

Explanation.—$\frac{1}{8}$ of 24 is 3, and $\frac{3}{8}$ are 9, the answer.

18. A gentleman owns $\frac{3}{11}$ of a farm worth \$1637; what is the value of his part? Ans. 446 $\frac{5}{11}$.

19. A owns $\frac{1}{3}$ of a ship, B $\frac{2}{5}$ as much, and C $\frac{3}{11}$ as much as B. What part of the ship does B own;—also, what is C's part?

Ans. { B's $\frac{2}{15}$.
C's $\frac{2}{55}$.

20. If the ship mentioned in the last question, is worth 10000 dollars, what is the value of A, B, and C's parts respectively?

Ans. { A's part is worth \$3333$\frac{1}{3}$.
B's " \$1333$\frac{1}{3}$.
C's " \$363$\frac{7}{11}$.

21. A gentleman divided 3 shares in a turnpike equally between 5 sons. If a share be worth 75 dollars, what is the value of each son's part?

Ans. \$45.

Note 3.—If a mixed number be given, it may be reduced to an improper fraction, and worked in every respect as a proper fraction, and for the same reason.

22. What is $\frac{3}{10}$ of 91$\frac{1}{2}$?

91$\frac{1}{2}$=$\frac{183}{2}$ and $\frac{3}{10}$ of $\frac{183}{2}$=$\frac{549}{20}$, or 27$\frac{9}{20}$.

23. A man sold 11 bushels of grain for 8$\frac{3}{4}$ dollars; what did he receive per bushel?

Explanation.—$\frac{1}{11}$ of the money he received for the whole will be the price of one bushel.

Ans. $\frac{35}{44}$ of a dollar.

24. If 14 bushels of salt cost 9$\frac{1}{14}$ dollars, what is it per bushel? Ans. $\frac{127}{196}$ of a dollar.

25. If a man buy 13 gallons of gin for $9\frac{7}{9}$ dollars, what does he give per gallon?

Ans. $\frac{88}{117}$ of a dollar.

26. If a yard of broadcloth cost five and three-eighths dollars, what will one-third of a yard cost?

Ans. \$$1\frac{19}{24}$.

27. A man gave $13\frac{2}{3}$ dollars for 8 sheep; what did he give a piece? Ans. \$$1\frac{17}{24}$.

28. If two acres of land are sold for $56\frac{13}{17}$ dollars, what will $\frac{1}{5}$ of one acre cost at the same rate?

Ans. \$$5\frac{23}{34}$.

29. A man bought 3 horses for $376\frac{3}{20}$ dollars, and 5 sheep for $\frac{1}{20}$ the value of 1 horse. What did the 5 sheep cost him; what is the cost of one sheep;—also, what was the average price of the horses?

Ans. { Average price of the horses, \$$125\frac{23}{60}$.
Cost of 5 sheep, \$$6\frac{323}{1200}$.
Cost of 1 sheep, \$$1\frac{1523}{6000}$.

30. What is $\frac{1}{3}$ of $\frac{1}{17}$ of $11169\frac{3}{10}$?

Ans. $219\frac{1}{170}$.

31. A merchant bought $\frac{1}{17}$ of a stock of goods, valued at 10809 dollars, and sold $\frac{1}{3}$ of his share at an advance of $\frac{1}{30}$ of the cost. What did he receive for the part he sold? Ans. \$$219\frac{1}{170}$.

32. What part of a penny is $\frac{1}{15}$ of a shilling?

Explanation.—$\frac{1}{15}$ of a shilling is $\frac{1}{15}$ of 12 pence, and the answer $\frac{12}{15}$, or $\frac{4}{5}$ of a penny. By multiplying the numerator by 12, the fraction expresses 12 fifteenths instead of 1 fifteenth. Now if we still call it a fraction of a shilling, the real value of the fraction is increased 12 fold. But the object of the question is, not to alter the real value of the fraction, and as we have made it a greater part of an unit, we must make it a fraction of an unit as much smaller than a shilling, as the fraction is greater now than it was before. A penny is the 12th part of a shilling; $\frac{12}{15}$ is 12 times as large as $\frac{1}{15}$, therefore, $\frac{12}{15}$ of a penny is equal to $\frac{1}{15}$ of a shilling. Any fraction may be made a fraction of a less denomination without altering its real value, by making it a fraction of as many units as it takes of the less denomination to equal 1 in that denomination in which the fraction is given, which is only multiplying the numerator by so many of the next less denomination.

33. What fraction of a shilling is equal to $\frac{1}{15}$ of a pound?

20 shillings equal 1 pound, then $\frac{1}{15}$ of 20 is the number required:

Ans. $\frac{20}{15}$, or $1\frac{5}{15}$, or $1\frac{1}{3}$ shillings.

34. What part of a farthing is $\frac{1}{1000}$ of a pound?

Ans. $\frac{24}{25}$.

35. What part of a minute, is $\frac{1}{725}$ of an hour.

Ans. $\frac{12}{125}$.

36. How many minutes in $\frac{1}{4}$ of a day?

Ans. 360.

37. How many pounds in $\frac{1}{6}$ of a ton, neat weight?

Ans. $333\frac{1}{3}$.

38. If a merchant sell $\frac{1}{7}$ of a hundred of sugar, neat, for 1 cent an ounce, what does he receive for the quantity sold?

Explanation.—The fraction reduced to the fraction of an ounce, will express the answer.

Ans. $2,28$\frac{4}{7}$.

39. In $\frac{3}{5}$ of an acre, how many square inches?

Ans. 3763584.

40. What part of 8 is $\frac{1}{3}$ of 9?

Explanation.—$\frac{1}{3}$ of 9$=\frac{9}{3}$, and $\frac{9}{3}=3$. 1 is $\frac{1}{8}$ of 8, and 3 must be $\frac{3}{8}$ of 8. We shall find whether $\frac{3}{8}$ of 8 be equal to $\frac{1}{3}$ of 9 by reducing both to a simple form: thus, $\frac{1}{3}$ of $9=\frac{9}{3}$, and $\frac{3}{8}$ of $8=\frac{24}{8}$, both equal to 3. The form of the compound fraction will be thus, $\frac{1}{3}$ of $\frac{9}{1}$ of $\frac{1}{8}$.

Ans. $\frac{3}{8}$.

41. What part of 2 equals $\frac{2}{3}$ of 3?

Ans. The whole.

42. What part of 3 equals $\frac{2}{5}$ of 10? Ans. $1\frac{1}{3}$.

Explanation.—The scholar must not infer from the answer that $1\frac{1}{3}$ equal $\frac{2}{5}$ of 10, for $\frac{2}{5}$ of 10 are 4, and the answer implies that 3 must be taken once and $\frac{1}{3}$ of it again, to make $\frac{2}{5}$ of 10.

43. What part of 6 is $\frac{3}{7}$ of 2? Ans. $\frac{1}{7}$.
44. What part of 1 is equal to $\frac{1}{18}$ of 7? Ans. $\frac{7}{18}$.
45. What part of 100 is $\frac{1}{11}$ of 1000? Ans. $\frac{10}{11}$.
46. What part of 15 is 3? Ans. $\frac{1}{5}$.
47. What part of 35 is 2? Ans. $\frac{2}{35}$.
48. A man sold 4 yards from a piece of cloth containing 38 yards. What part of the piece did he sell? Ans. $\frac{1}{7}$.
49. A merchant bought a hogshead of molasses for 36 dollars, and sold enough of it to come to 20 dollars at the same rate. What part of the hhd. did he sell? Ans. $\frac{5}{9}$.
50. $\frac{4}{17}$ of a ship's cargo valued at 7000 dollars, was thrown overboard in a storm. What was A's loss who owned $\frac{4}{11}$ of the cargo?
Ans. $\$598\frac{174}{187}$.
51. In a certain bridge there are 50 shares, and are owned equally between 7 men. Now if A and B sell $\frac{1}{3}$ of their shares to an eighth person, H, how many shares, and also what proportion of the whole bridge will H own?
Ans. $4\frac{16}{21}$ shares, or $\frac{2}{21}$ of the whole bridge.
52. What part of a shilling is $\frac{1}{15}$ of a penny?
This question is the reverse of question 32d.

Explanation.—$\frac{1}{15}$ of a penny is $\frac{1}{15}$ of $\frac{1}{12}$ of a shilling, which is equal to $\frac{1}{180}$ of a shilling. Multiplying the denominator by as many of the less denomination as are required to equal 1 in the greater, changes the fraction to a fraction of the greater, without altering its value, because we lessen the fraction as much as the unit is increased.

53. What part of a foot is $\frac{1}{7}$ of an inch?
Ans. $\frac{1}{84}$.

54. What part of a rod is $\frac{2}{5}$ of an inch?
Ans. $\frac{1}{495}$.

55. What part of a year is $\frac{3}{13}$ of an hour?
$\frac{3}{113880}$.

Remark.—The year is estimated at 365 days in the last question and in the 59th.

56. If a person draw 3 pints from a hhd. of wine, what part of the hhd. does he take away?
Ans. $\frac{1}{168}$.

57. A man sells 59 solid feet of wood; what part of 4 cords does he sell? Ans. $\frac{59}{512}$.

58. What part of a week is 1 day and 5 hours?

Explanation.—The denominator being equal to an unit 1 of that denomination to which the fraction belongs, 1 of that denomination reduced to the lowest denomination contained in the given numbers, must be made the denominator, and the given numbers reduced to the same denomination, a numerator.

To express the whole time mentioned in the last question in hours, the one day must be reduced to hours, and 5 hours added. (See note page 119.) One day and 5 hours equal 29 hours. To know what part of a week 29 hours are, one week must be reduced to hours and made the denominator of the fraction. One week equals 168 hours, and 29 hours must be $\frac{29}{168}$ of a week.

59. What fraction of a year is 4 minutes?
Ans. $\frac{1}{131400}$.

60. What part of a pound is 2 s. 6 d.?
Ans. $\frac{30}{240}=\frac{1}{8}$.

61. What part of a yard is 3 qr. 1 na.?
Ans. $\frac{13}{16}$.

62. A merchant sells $4\frac{3}{4}$ yards from a piece of cloth measuring 31 yards. What part of the piece does he sell? Ans. $\frac{19}{124}$.

63. If the moon pass through $13^{\circ}\ 10'\ 30''$ of her

orbit in one day, what part of her revolution does she perform in the same time? Ans. $\frac{14743}{129600}$.

64. A man owning 1 A. 3 R. 31 r. of land, laid out 56 r. of it for a garden. What part of the whole is the garden? Ans. $\frac{56}{311}$.

To find a common denominator.

Fractions have a common denominator when the denominators are all equal to each other. $\frac{2}{15}$, $\frac{4}{15}$, $\frac{7}{15}$, are fractions having a common denominator; but $\frac{1}{7}$, $\frac{4}{15}$, and $\frac{3}{11}$, have not a common denominator.

RULE.

Multiply all the denominators together, the product will be a common denominator. Multiply each numerator by all the denominators except its own; the product written over the common denominator will form a fraction expressing the same part of an unit, as the fraction whose numerator was multiplied.

1. Change $\frac{1}{2}$, $\frac{3}{4}$, $\frac{2}{3}$ to fractions having a common denominator.

Operation.

```
      1/2  3/4  2/3
      ————
             2  —
             4  12
        3   —    2
   1    3    8  —
   4   —     2  24  common denominator.
  —     9   —
   4    2   16  new numerator for 2/3=16/24.
   3   —
  —    18  new numerator for 3/4=18/24.
  12   new numerator for 1/2=12/24.
```

Given fractions $\frac{1}{2}$, $\frac{3}{4}$, $\frac{2}{3}$.

The same fractions reduced to a common denominator $\frac{12}{24}$, $\frac{18}{24}$, $\frac{16}{24}$.

Illustration.—When all the denominators are multiplied together, the product is a denominator expressing an unit divided into a greater number of parts; and multiplying each numerator by all the denominators except its own, the product will be as much less than the product of the denominators, as that numerator is less than its own denominator, and will be the same part of the common denominator, as the given numerator is of the given denominator.

Let it be required to change $\frac{1}{3}$ and $\frac{1}{4}$ to fractions having a common denominator. To obtain the new denominator, we multiply the 3 by 4, and the product is 12. To obtain a new numerator for $\frac{1}{3}$, we multiply its numerator by 4, increasing it by the same number that we did the denominator, and the result is $\frac{4}{12}$. To change $\frac{1}{4}$, we multiply both denominator and numerator by 3, and the result is $\frac{3}{12}$. 4 is the same part of 12 that 1 is of 3, and 3 is the same part of 12 that 1 is of 4; therefore, $\frac{4}{12}=\frac{1}{3}$, and $\frac{3}{12}=\frac{1}{4}$.

The scholar will perceive by examining the operations of the following examples, that when more than two fractions are given, multiplying as directed by the rule, we increase the numerator and denominator of the same fractions, by the same numbers.

2. Change $\frac{2}{9}$, $\frac{3}{7}$, and $\frac{4}{14}$, to fractions having a common denominator.

Ans. $\frac{196}{882}$, $\frac{378}{882}$, and $\frac{252}{882}$.

3. Change $\frac{6}{19}$ and $\frac{75}{13}$ to fractions having a common denominator. Ans. $\frac{78}{247}$ and $\frac{1425}{247}$.

4. Change $\frac{11}{17}$ and $20\frac{1}{18}$ to fractions having a common denominator. Ans. $\frac{198}{306}$ and $20\frac{17}{306}$.

Note 4.—Mixed numbers may be brought to improper fractions before the operation, or the whole number omitted and set before the same fraction after the operation, as the nature of the question may require.

5. If $\frac{1}{29}$ and $\frac{3}{28}$ of a dollar be expressed by parts of a dollar equally divided, what fractions will express both quantities? Ans. $\frac{28}{812}$ and $\frac{87}{812}$.

6. A owns $\frac{1}{4}$ of a stage and D $\frac{3}{13}$; which owns the greatest part? Ans. A.

Explanation.—After a common denominator is found, the numerators will show which is the greatest fraction.

ADDITION OF VULGAR FRACTIONS.

Illustration.—When a single thing of any kind is divided into several equal parts, or two equal things are so divided, any number of those parts may be added together in the same manner as simple numbers. If a hogshead of wine be divided into 63 equal parts, and one man have 9 of those parts, and another 8, both would have 17 parts, the sum of 9 and 8; or, in other words, both men would possess 17 gal. of wine; for, as 63 gal. equal a hhd. $\frac{1}{63}$ part of a hhd. is one gal. The same expressed in a fractional form is thus, $\frac{9}{63}+\frac{8}{63}=\frac{17}{63}$, adding the numerators only. It is necessary to write the common denominator under those parts owned by each man, to show that the parts owned by one, are just as large as those owned by the other. But if we suppose one hhd. of wine divided into 63 equal parts, and one man to possess 9 of these parts, and another hhd. divided into 4 equal parts, and another person to possess one of those parts, we could not add the parts owned by both persons as they stand, since 1 quarter of a hhd., or $\frac{1}{4}$, is larger than 1 gal., or $\frac{1}{63}$. Let the fractions expressing the parts owned by both persons, $\frac{9}{63}$ and $\frac{1}{4}$, be reduced to a common denominator, they will be $\frac{36}{252}$ and $\frac{63}{252}$. After the fractions are reduced to a common denominator, each hhd. is supposed divided into 252 equal parts, and the person who owns $\frac{1}{4}$ of a hhd. will own 63 of 252 parts, (as may be seen by dividing 252, which denotes the whole hhd. by 4,) and the other 36 of the 252 equal parts. Each fractional unit in 36 and 63, expresses the same part of a hhd., $\frac{1}{252}$, therefore, the sum of 36 and 63 will denote the number of parts owned by both persons.

We may have two or more fractions of equal denominators which cannot be added together until they are changed in certain respects. We cannot add $\frac{1}{5}$ of a pound and $\frac{1}{5}$ of a shilling together, for their sum $\frac{2}{5}$, would be neither $\frac{2}{5}$ of a pound, nor $\frac{2}{5}$ of a shilling. Let the $\frac{1}{5}$ of a shilling, be brought to the fraction of a pound. (See *Explanation* to question 52d, page 152,) and it is $\frac{1}{100}$ of a pound; then $\frac{1}{5}$ and $\frac{1}{100}$ are fractions of a pound, and may be added after they are reduced to a common denominator. Several fractions of the same, or of equal units, having equal denominators, express as many fractional units of the same unit or quantity, as there are ones in all the numerators; therefore, the sum of the numerators written over the common denominators, will express the same part of an unit as is denoted by the given fractions.

RULE.

After preparing the fractions as explained in the illustration, add the numerators and write the sum over one of the denominators.*

1. How many eighths in $\frac{3}{8}$, $\frac{1}{8}$ and $\frac{2}{8}$? Ans. 6.
2. What is the sum of $\frac{3}{13}$, $\frac{4}{13}$ and $\frac{5}{13}$? Ans. $\frac{12}{13}$.
3. What is the sum of $\frac{1}{7}$, $\frac{2}{7}$ and $6\frac{3}{7}$? Ans. $6\frac{6}{7}$.
4. What is the sum of $\frac{1}{15}$, $\frac{6}{15}$ and $\frac{31}{15}$?
Ans. $2\frac{8}{15}$.
5. What is the sum of $\frac{3}{8}$ and $\frac{1}{3}$? Ans. $\frac{17}{24}$.

---

* All fractions in answers after this, will be reduced to their lowest terms.

6. What is the sum of $\frac{1}{19}$, $\frac{2}{3}$ and $\frac{7}{4}$?

Ans. $2\frac{107}{228}$.

7. A merchant purchased a lot of cotton yarn, and sold $\frac{3}{11}$ of it to one man, and $\frac{2}{7}$ of $\frac{1}{4}$ to another: what part of the lot did he sell?* Ans. $\frac{53}{154}$.

8. A man sells $\frac{3}{5}$ of a lot of iron to one man, and $\frac{1}{2}$ of $\frac{4}{5}$ to another: what part does he sell?

Ans. The whole.

9. If one gentleman own $\frac{3}{21}$ of a common pasture: a 2d, $\frac{27}{35}$: and a third, $\frac{2}{29}$, what part of the pasture is owned by the 3? Ans. $\frac{998}{1015}$.

10. What part of a day is the sum of $\frac{3}{10}$ of a day and $\frac{3}{11}$ of an hour? Ans. $\frac{137}{440}$.

Explanation.—$\frac{3}{11}$ of an hour equal $\frac{3}{264}$ of a day.

11. What part of a pound is the sum of $\frac{1}{7}$ of a pound, and $\frac{1}{7}$ of a shilling? Ans. $\frac{3}{20}$.

12. What part of a ton is the sum of $\frac{3}{7}$ of a cwt. and $\frac{2}{9}$ of a qr.? Ans. $\frac{61}{2520}$.

13. What part of an inch is the sum of $\frac{6}{3564}$ of a rod and $\frac{3}{1140480}$ of a mile? Ans. $\frac{1}{2}$.

14. What part of a pint is the sum of $\frac{1}{8}$ of a quart and $\frac{1}{11}$ of a gal.? Ans. $\frac{43}{44}$.

15. A merchant has 4 casks of rum which contain No. 1, $97\frac{3}{4}$ gal.; No. 2, $100\frac{4}{5}$; No. 3, $110\frac{3}{7}$; and No. 4, $115\frac{2}{9}$. How many gallons in the 4 casks?

Operation. 945

$97\frac{3}{4} = 97\frac{945}{1260}$ 1008

$100\frac{4}{5} = 100\frac{1008}{1260}$ 540

$110\frac{3}{7} = 110\frac{540}{1260}$ 280

$115\frac{2}{9} = 115\frac{280}{1260}$

2773 sum of the numerators.

$424\frac{253}{1260}$ Ans.

1260)2773(2 number of whole gal. contained
2520 in the fractions.

New num. 253

---

* *The* compound fraction must be reduced to a simple one, and *the simple* fraction used instead of the compound one.

Explanation.—In the first place, we reduce the fractions to a common denominator. Each fraction reduced, expresses the same part of a gallon which it did before. (See Illustration, page 156.) Each mixed number in the right hand column, represents the same quantity as the number standing against it in the left. Next we add all the numerators of the fractions in the right hand column, and find their sum to be 2773. It requires 1260 of the fractional units to equal one gal., therefore, as many times as 1260 can be taken from 2773, so many whole gallons are expressed by the fractions, which we find to be two, and these two gal. we add to the unit column.

In adding other mixed numbers, one must be added to the whole number for each time the common denominator can be taken from the sum of the numerators, for the common denominator will always show how many parts, like those expressed by the fractions, are required to equal an unit or 1, in a whole number.

16. A man owes 3 debts, viz.: one of $\$75\frac{22}{31}$; one of $\$28\frac{3}{28}$; and one of $\$43\frac{21}{23}$. What is the sum of the 3 debts? Ans. $\$147\frac{14571}{19964}$.

17. A, B and C, performed a journey in company with equal sums of money to defray the expenses of the same. A spent $\frac{3}{7}$ of his money, B $\frac{9}{11}$ of his, and C $\frac{58}{77}$ of his; what part of the money possessed by the 3, was expended? Ans. $\frac{2}{3}$.

18. Add together $6\frac{1}{11}$, $\frac{11}{2}$, 9, $5\frac{2}{3}$, and $\frac{4}{7}$.
Sum, $26\frac{383}{462}$.

* * *

MULTIPLICATION OF VULGAR FRACTIONS.

RULE.

Multiply the numerator of the multiplicand by the *numerator* of the multiplier, and the denominator of the multiplicand by the denominator of the multiplier,—the former product written over the latter, will constitute the answer. If either or both of the given fractions be compounded, the continued product of the numerators and the continued product of the denominators will form the answer required.

Remark. The product will be as much less than the multiplicand as the multiplier is less than an unit. If the multiplier be more than an unit, the product will be as much greater than the multiplicand as the multiplier is greater than an unit.

Most of the questions belonging to this rule are similar to the first 30 questions in reduction of compound fractions, and are operated in a similar manner and for the same reasons. The student should re-examine the illustration on page 145, and the several explanations under the first part of that rule, for he should pass over no questions without endeavouring to understand the principles on which their operations are founded.

1. What is the product of $\frac{1}{9}$ multiplied by $\frac{2}{7}$?

Explanation.—The multiplier $\frac{2}{7}$, being less than an unit, we can only take such a part of $\frac{1}{9}$, the multiplicand, as $\frac{2}{7}$ is of an unit, and this we shall do by multiplying one fraction by the other, since the product of the denominators will be greater than that of the numerators. The question is the same as if it had been—what is $\frac{2}{7}$ of $\frac{1}{9}$.

$\frac{1}{9}\times\frac{2}{7}=\frac{2}{63}$ the answer.

2. Multiply $\frac{21}{31}$ by $\frac{3}{16}$. Prod. $\frac{63}{496}$.
3. What is the product of $\frac{13}{14}$, multiplied by $\frac{1}{6}$ of $\frac{2}{3}$? Ans. $\frac{13}{126}$.
4. What is the product of $\frac{139}{8}$ into $\frac{1}{12}$? Ans. $\frac{139}{96}$, or $1\frac{43}{96}$.

Note 5.—When a mixed number is given, it is most convenient to change it to an improper fraction, then the improper fraction may be used instead of the mixed number, the same as the improper ction in the last question.

5. If a man own $\frac{1}{5}$ of 96$\frac{3}{4}$ acres of land, how many acres does he own ? Ans. $\frac{387}{20}$, or 19$\frac{7}{20}$.

6. At 3 $\frac{5}{7}$ dollars per cwt., what will $\frac{2}{5}$ of a cwt. of logwood come to ? Ans. 1\frac{17}{35}$.

7. If a pipe of brandy be worth 96\frac{3}{25}$, what is $\frac{4}{11}$ of a pipe worth ? Ans. 34\frac{262}{275}$.

8. What is the freight of a quantity of goods which cost 2673\frac{7}{9}$, if the freight equal $\frac{4}{11}$ of $\frac{1}{4}$ of the first cost ? Ans $243 $\frac{7}{99}$.

9. At 5$\frac{1}{4}$ dollars per cwt. what will 12$\frac{3}{4}$ cwt. of iron cost ?

Explanation.—Reduce both numbers to improper fractions.

Ans. 66\frac{15}{16}$.

10. If a barrel of mackerel cost 4\frac{11}{12}$, what will 3$\frac{1}{2}$ bar. cost at the same rate ?

Ans. 17\frac{5}{24}$.

11. What is the product of 120$\frac{1}{2}$, multiplied by 100$\frac{3}{5}$? Ans. 12122$\frac{3}{10}$.

12. A man labours for 12$\frac{1}{3}$ dollars per month, what will be his wages for 8$\frac{2}{3}$ months ?

Ans. 106\frac{8}{9}$.

13. At $\frac{1}{5}$ of a dollar per yard, what will 4 yards of riband come to ?

Explanation.—If one yard cost 1 fifth of a dollar, 4 yards must cost 4 fifths ; and this we shall obtain by multiplying the numerator of $\frac{1}{5}$ by 4 the number of yards. Ans. $\frac{4}{5}$ of a dollar. The numerator of any simple fraction always denotes how many parts of an unit, or single quantity, are expressed by the fraction, therefore by multiplying the numerator by a whole number, we place in the numerator of the product, the number of parts, or fractional units, contained in the given numerator, as many times as there are units in the whole number. In multiplying the numerator of a fraction by a whole number, we increase the real value of the fraction, because the denominator not being altered, the parts of a quantity expresed by the numerator of the product, are just as large as the parts contained in the numerator of the multiplicand, and we have more of them.

14. What is the product of $\frac{9}{13}$, multiplied by 2? Ans. $\frac{18}{13}$, or $1\frac{5}{13}$.

15. What number is equal to 5 times $\frac{2}{5}$? Ans. 2.

16. At $\frac{1}{7}$ of a shilling per pound, what will 7 lb. of copperas come to? Ans. 1 s.

17. If a man work for $\frac{11}{14}$ of a dollar per day, what will he receive for 101 days' work? Ans. $79\frac{5}{14}$.

18. If a horse eat $\frac{3}{11}$ of a bushel of oats in one day, how many bushels will he eat in 30 days? Ans. $8\frac{2}{11}$.

19. What will a hog, weighing 371 lb. come to at $\frac{1}{23}$ of a dollar per pound? Ans. \$$16\frac{3}{23}$.

20. The toll of a certain bridge is \$825 a year, and is divided into 60 shares. A gentleman owning 7 shares, wishes to know what his part of the toll will be for one year.

Explanation.—He owns $\frac{7}{60}$ of the whole. If we divide 825 by 60, the quotient will be the value of one share, and multiplying the value of one share by 7, will give the value of 7 shares. But if we multiply 825 by 7, and divide the product by 60, the dividend, 5775 is 7 times 825, consequently the quotient must be 7 times as large as when we divided 825 by 60. Ans. \$$96\frac{1}{4}$. From this explanation it appears, that we shall obtain the same result whether we multiply a whole number by the numerator of a fraction, and divide by the denominator, or divide the whole number by the denominator and multiply the quotient by the numerator.

21. What is the product of 1091, multiplied by $\frac{7}{9}$? Ans. $848\frac{5}{9}$.

22. A and B bought a lot of cotton in company for 150 dollars, of which A paid 635 dollars, and B the

remainder. They sell the same for 1425 dollars; what must A receive; also, what is B's gain?

Ans. $\left\{\begin{array}{ll}\text{A receives} & \$786\frac{38}{48}.\\ \text{B's gain} & \$123\frac{7}{48}.\end{array}\right.$

23. 7 men and a boy own a house in company. Each man's share is worth $480, and the boy's share is equal to $\frac{1}{3}$ of one man's. What is the value of the house? Ans. $3520.

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SUBTRACTION OF VULGAR FRACTIONS.

Illustration.—When two fractions of the same unit or quantity, or of two equal units or quantities, have equal denominators, a fractional unit, (that is, one of the parts contained in the numerator,) in one numerator, is equal to a fractional unit in the other, and the difference between the numerators will be the difference between the parts of an unit or a quantity expressed by one fraction, and the parts expressed by the other. If we have $\frac{2}{12}$ and $\frac{5}{12}$ of a dollar, the 5 twelfths express $\frac{3}{12}$ more than the 2 twelfths.

The same may be expressed as follows: let a dollar be divided into 12 equal parts, and let A have 5 of these parts and B 2. A must have 3 parts more than B.

Fractions having unequal denominators cannot be subtracted until they are reduced to a common denominator. The difference between $\frac{3}{7}$ and $\frac{4}{8}$, is neither $\frac{1}{7}$ nor $\frac{1}{8}$. $\frac{3}{7}$ equal $\frac{24}{56}$, and $\frac{4}{8}$ equal $\frac{28}{56}$, and the difference is $\frac{4}{56}$; but $\frac{1}{7}$ equals $\frac{8}{56}$, and $\frac{1}{8}$ equals $\frac{7}{56}$.

RULE.

Prepare the fractions as in addition.

Subtract the less numerator from the greater, and write the difference over one of the denominators.

1. What is the difference between $\frac{3}{5}$ and $\frac{2}{5}$?
$\frac{3}{5}-\frac{2}{5}=\frac{1}{5}$ the answer.

2. What is the difference between $\frac{21}{47}$ and $\frac{36}{47}$?
Ans. $\frac{15}{47}$.

3. Subtract $\frac{1}{2}$ from $\frac{5}{6}$. Ans. $\frac{4}{12}=\frac{2}{6}=\frac{1}{3}$.

4. A and B travel together; A pays $\frac{5}{13}$ of the expense, and B $\frac{16}{26}$; what part of the expense does B pay more than A? Ans. $\frac{3}{13}$.

5. Subtract $\frac{4}{9}$ from $\frac{5}{6}$ of $\frac{11}{12}$. Rem. $\frac{23}{72}$.

Explanation.—$\frac{5}{6}$ of $\frac{11}{12}=\frac{55}{72}$, which must be used instead of the compound fraction.

6. What is the difference between $\frac{2}{23}$ and $\frac{71}{3}$?
Ans. $23\frac{40}{69}$.

7. If one man own $\frac{1}{11}$ of $\frac{2}{3}$ of a ship, and another, $\frac{2}{21}$ of $\frac{3}{4}$, what is the difference of their shares?
Ans. $\frac{5}{462}$.

8. Subtract $\frac{11}{7}$ from $\frac{13}{8}$. Rem. $\frac{3}{56}$.

9. If a man pay $29 towards a debt of 46\frac{5}{8}$, what will remain unpaid? Ans. 17\frac{5}{8}$.

10. How much more brandy in that cask which contains 67$\frac{3}{16}$ gal. than in that which contains 51$\frac{3}{16}$ gal.? Ans. 16.

11. What number is equal to the difference between 58$\frac{3}{17}$ and 60$\frac{1}{2}$?

Explanation.—The easiest method of working questions like this, is to reduce the fractions to a common denominator by themselves, without bringing the mixed numbers to improper fractions. Care should be taken that the same fraction which is taken from one whole number after being reduced, should be placed to the right hand of the same whole number. The numbers after the fractions are reduced to a common denominator, will be 58$\frac{6}{34}$ and 60$\frac{17}{34}$.
Ans. 2$\frac{11}{34}$.

12. If 4$\frac{5}{6}$ gal. of gin be drawn from a cask containing 67$\frac{7}{8}$ gal., how many gallons will remain in the cask? Ans. 63$\frac{1}{24}$.

13. How much larger is $9\frac{1}{4}$ than $7\frac{3}{4}$?

Operation 1.	Operation 2.
$9\frac{1}{4}$	$8\frac{5}{4}$
$7\frac{3}{4}$	$7\frac{3}{4}$
Ans. $1\frac{2}{4}=1\frac{1}{2}$	Ans. $1\frac{2}{4}=\frac{1}{2}$

Explanation.—In operation 1, we suppose 4 the denominator added to 1 the numerator of the upper fraction, and subtract the numerator of the under fraction from 5 their sum. By adding 4 to the upper numerator, we increase the upper number by 1, for the denominator equals 1. To balance this 1 we add 1 to 7, the whole number in the subtrahend. In operation 2, 1 is taken from 9 and supposed divided into 4 equal parts, and these 4 parts added to the upper numerator, which makes the upper fraction the largest. The subtracting can now be performed as in the preceding examples, and the remainder is found to be the same as in the first operation. The reason for not adding 1 to the 7 in operation 2, is because we have not increased the upper number.

14. A man owing $100\frac{1}{5}$ dollars, paid all but $15\frac{3}{7}$ dollars, how much did he pay? Ans. $84\frac{27}{35}$.

15. What is the difference between $1\frac{1}{4}$ and $\frac{3}{4}$? Ans. $\frac{1}{2}$.

16. A farmer sells $11\frac{3}{5}$ bushels of grain from a chest containing 17 bushels; how much remains?

$$\begin{array}{r} 17 \\ 11\frac{3}{5} \\ \hline 5\frac{2}{5} \text{ bus.} \end{array}$$

Explanation.—There being no fraction in the larger number, we subtract the numerator from the denominator, which is the same as if we had set 5 fifths in the upper number and taken 3 fifths from them: but adding 5 fifths to 17 would make 18 in the minuend, therefore 1 must be added to 11 the whole number in the subtrahend.

17. A man sells $19\frac{8}{13}$ acres from a farm containing 100; how many acres remain? Ans. $80\frac{5}{13}$.

18. What number must be added to $88\frac{1}{21}$, so that the sum shall be 96? Ans. $7\frac{20}{21}$.

19. What fraction of a pound is equal to the difference between $\frac{2}{3}$ of a pound and $\frac{2}{3}$ of a shilling?

Explanation.—First reduce the fraction of a shilling to the fraction of a pound, (See Explanation page 152,) then proceed as with other fractions?

Ans. $\frac{19}{30}$.

20. What part of an hour is the difference between $\frac{1}{30}$ of a day and $\frac{1}{7}$ of an hour? Ans. $\frac{23}{35}$.

DIVISION OF VULGAR FRACTIONS.

To divide a whole number by a fraction.

RULE.

Multiply the whole number by the denominator and divide the product by the numerator.

1. How many times can $\frac{1}{4}$ be subtracted from 3?

```
       3
       4
     ———
 1 ) 1 2
     ———
Ans. 1 2
```

Illustration.—1 fourth can be taken from 1 fourth once, and from 4 fourths 4 times, but 4 quarters equal an unit, or 1. If $\frac{1}{4}$ can be taken 4 times from 1 unit, it can be taken 3 times 4, or 12 times, from 3 units. Multiplying 3 by 4 reduces the whole number to fourths, for we place 4 in the product once for each unit in the whole number, as was explained in multiplication of simple numbers. The question is really no other than this,—How many times can 1 fourth be taken from 12 fourths. Had the divisor been $\frac{3}{4}$ instead of $\frac{1}{4}$, the divisor would have been 3 times as large and the quotient could have been only $\frac{1}{3}$ of what it was before, viz. 4 instead of 12. Multiplying the whole number by the denominator of any simple fraction reduces the whole number into the same parts of an unit as those expressed by the fraction, and the numerator being composed of parts of an unit equally small, the quotient must express the number of times the parts contained in the numerator can be taken from the number of parts contained in the whole number. If the denominator be 8, multiplying the whole number by 8 the product will express each unit in the whole number divided into 8 equal parts. Should the divisor be an improper fraction, and the numerator contain more units than the dividend, the quotient must be less than the dividend; but if the divisor be less than 1, which is the case when it is a proper fraction, the quotient will be greater than the dividend.

It may at first appear singular that the quotient should ever be greater than the dividend. This apparent mystery vanishes when the student reflects, that a part of one can be taken from 1 more than once.

2. How many times can $\frac{2}{3}$ be taken from 6?

$6\times3\div2=9$ the answer.

3. How many times can $\frac{4}{5}$ be subtracted from 12?

Ans. 15.

4. What is the quotient of 20, divided by $\frac{11}{13}$?

Ans. $23\frac{7}{11}$.

5. If a merchant sell cloth to the amount of 66 dollars at $\frac{3}{5}$ of a dollar per yard, how many yards does he sell? Ans. 110.

6. A flock of sheep was sold for 100£. the price of each sheep being $\frac{10}{19}$ of a pound, how many sheep in the flock? Ans. 190.

7. If a yoke of oxen eat $\frac{2}{7}$ of a bushel of corn in one day, how long will 21 bushels last them?

Ans. $73\frac{1}{2}$ days.

Note 6.—If the fraction be a compound one, reduce it to a simple one.

8. At $\frac{5}{6}$ of $\frac{3}{4}$ of a dollar per bushel, how many bushels of corn can be bought for 125 dollars?

Ans. 200.

9. What is the quotient of 1001, divided by $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$? Ans. 4004.

Note 7.—It was shown in the illustration, that a whole number might be divided by an improper fraction in the same manner as by a proper fraction. An improper fraction obtained by changing a mixed number to an improper fraction, is equal to the mixed number.

10. What is the quotient of 114, divided by $6\frac{1}{3}$?

```
  6⅓              1 1 4
  3                   3
 ——              ———————
 1 9     1 9 ) 3 4 2 ( 1 8 Ans.
               1 9
             ———————
               1 5 2
               1 5 2
             ———————
```

Explanation.—In $6\frac{1}{3}$ there are 19 thirds, which is the divisor; in 114 there are 342 thirds.

11. If $2\frac{1}{8}$ yards of broadcloth make 1 coat, how many coats will 34 yards make? Ans. 16.

12. How many square yards of floor will 27 yards of carpeting cover, if it require $1\frac{1}{3}$ yards of carpeting to cover 1 square yard of floor? Ans. $20\frac{1}{4}$.

13. A gentleman has $9000 deposited in a certain bank, which is $\frac{3}{7}$ of his whole estate. What is the amount of his property? Ans. $21000.

14. Sold a quantity of cheese for 56 dollars, at $\frac{2}{19}$ of a dollar per pound; how many pounds of cheese were sold. Ans. 532.

To divide a fraction by a fraction.

Illustration.—One is placed in the quotient every time the divisor is taken from the dividend. Let it be required to divide $\frac{3}{4}$ by $\frac{1}{4}$. This may be done as follows, $\frac{1}{4}$) $\frac{3}{4}$ ($\frac{3}{1}=3$, for dividing the numerator by 1 does not lessen it. $\frac{1}{4}$ can be taken from $\frac{3}{4}$

as many times as 1 unit can be taken from 3 units. When the divisor and dividend have a common denominator, and the numerator of the dividend can be divided by the numerator of the divisor without a remainder, we may divide as above and the quotient of the numerators will be the true quotient. When the denominators are equal, dividing the numerators is dividing several parts of the unit or quantity by another number of parts equally small, and the denominators may be entirely omitted in the operation. If it be required to divide $\frac{4}{5}$ by $\frac{2}{5}$, the numerators may be considered as whole numbers, and divided as such, and the quotient would be 2, a whole number. If the divisor and dividend have not a common denominator, the quotient may be obtained without reducing them in the common way.

Let $\frac{4}{10}$ be divided by $\frac{1}{5}$.

$\frac{1}{5}$ $\frac{4}{10}$
5

50 com. denom.

1 4
10 5

$\frac{10}{50}=\frac{1}{5}$ $\frac{20}{50}=\frac{4}{10}$

10)20(2 Ans.
20

We reduce the fractions in this example to a common denominator for the purpose of showing more plainly how the numerators may be obtained without their denominators. $\frac{10}{50}=\frac{1}{5}$, and $\frac{20}{50}=\frac{4}{10}$. After the fractions are reduced, all we have to do, is to divide the numerator of the dividend $\frac{20}{50}$ by the numerator of the divisor $\frac{10}{50}$, which gives a quotient of 2. To get 10 fiftieths, we multiply 1 the numerator of the divisor by 10 the denominator of the dividend; and to obtain 20 fiftieths, we multiply 4 the numerator of the dividend by 5 the denominator of the divisor. *It will be* seen that in dividing we have nothing to do with 50 the common denominator. As the denominator is not used in dividing, all that is necessary to be done in this example, is to multiply each given numerator by the denominator of the other fraction, which gives the numerators the same as when the common denominator is found. The most convenient method of obtaining the numerators for dividing without the denominators, is, to let the terms of the divisor change places, that is, setting the denominator above the line and the numerator below it, (which is generally called inverting the divisor,) and then multiply the two numbers above the line together, and the two below the line. The two numerators in the last example may be found in this way as follows; $\frac{5}{1} \times \frac{4}{10} = \frac{20}{10} = 2$.

Multiplying 4 by 5, which is the denominator of the divisor, gives 20 the dividend, or the numerator of the dividend; and multiplying 1 by 10 the denominator of the dividend, gives ten the divisor, but the dividend now stands in the place of the numerator, and the divisor in that of the denominator. 10, the denominator of $\frac{20}{10}$, is the numerator of the fraction which is the divisor; but as that numerator is used as the divisor, it properly becomes the denominator of a fraction when the dividend is made the numerator of the same fraction, as the denominator always represents a divisor. (See Illustration, page 145.) By putting the denominator of the divisor in place of the numerator, and the numerator in that of the denominator, then multiplying as in the last example, the products form a fraction having the numerator of the dividend uppermost, and that of the divisor in place of the denominator; and these two numbers will always be of the same denomination, for they are the two numerators, which would belong to the common denominator formed by the product of the two given denominators.

To obtain the quotient of the fractions in a single number, the numerator of the fraction, made by *multiplying*, must be divided by the denominator. If the *numerator* is less than the denominator, we do not actually divide one fraction by the other, and can only represent the quotient by the fraction formed by multiplying.

The foregoing illustration contains the reasons for the following,

RULE.

Change the terms of the divisor, then multiply the upper numbers together for a numerator, and the under numbers for a denominator, the fraction thus obtained will express the quotient. If the numerator be equal to or greater than the denominator, divide the numerator by the denominator.

15. How many times can $\frac{1}{5}$ be subtracted from $\frac{3}{5}$?

$\frac{3}{5} \div \frac{1}{5} = \frac{3}{1}$, or by the rule $\frac{3}{5} \times \frac{5}{1} = \frac{15}{5} = 3$ the Ans.

16. Divide $\frac{5}{6}$ by $\frac{2}{3}$? Ans. $1\frac{1}{4}$.

17. What is the quotient of $\frac{2}{7}$, divided by $\frac{15}{17}$?

Explanation.—$\frac{2}{7}$ is less than $\frac{15}{17}$, and the quotient will be less than an unit. Ans. $\frac{34}{105}$.

18. At $\frac{2}{9}$ of a dollar per pound, how many pounds of coffee can be bought for $\frac{9}{11}$ of a dollar? Ans. $3\frac{15}{22}$.

19. If $\frac{3}{4}$ of a bushel of corn cost $\frac{2}{3}$ of a dollar, what will 1 bushel cost?* Ans. $\frac{8}{9}$ of a dollar.

20. If $\frac{1}{7}$ of a yard of cloth cost $\frac{3}{8}$ of a dollar, what is the cloth per yard? Ans. \$$2\frac{5}{8}$.

21. If several men own equal shares of $\frac{9}{11}$ of a ship's cargo, and each man's share is $\frac{1}{22}$ of the whole cargo, how many owners to the $\frac{9}{11}$. Also, if each man's share be worth \$500, what is the value of the whole cargo?

Ans. { 18 owners to the $\frac{9}{11}$
Value of the cargo \$11000.

---

* The price of a bushel must be as much larger than $\frac{2}{3}$ as 1 is larger than $\frac{3}{4}$. Multiplying the numerator of $\frac{2}{3}$ by 4, while the other numerator is multiplied by 3, gives this *increase* in the quotient.

22. How many times $\frac{1}{7}$ in $\frac{8}{9}$? Ans. $6\frac{2}{9}$.
23. $\frac{11}{12}$ are how many times $\frac{1}{12}$? Ans. 11.
24. $\frac{11}{12}$ are how many times $\frac{1}{24}$? Ans. 22.
25. In $\frac{2}{3}$ how many $\frac{1}{4}$? Ans. $2\frac{2}{3}$.
26. How many times can $\frac{2}{5}$ be taken from $\frac{3}{11}$? Ans. $\frac{15}{22}$.
27. How many times can $\frac{3}{4}$ be taken from $\frac{1}{4}$? Ans. $\frac{1}{3}$.

Note 8.—The meaning of the answers in the two last questions, is, that such a part of the divisor can be taken from the dividend. Only $\frac{1}{3}$ of $\frac{3}{4}$ can be taken from $\frac{1}{4}$.

28. How many times can $\frac{1}{4}$ be taken from 7?

Explanation.—As the numerator of the divisor is but 1, we have only to multiply the whole number by the denominator. When the numerator is more than 1, and the dividend a whole number, let 1 be used as a denominator to the whole number, which will carry the numerator of the divisior to its proper place. (See Illus. page 168.) Ans. 28.

29. How many $\frac{3}{4}$ in 7? Ans. $9\frac{1}{3}$.
30. How many times $\frac{3}{14}$ in 14? Ans. $65\frac{1}{3}$.
31. How many times can $\frac{4}{13}$ be taken from 1? Ans. $3\frac{1}{4}$.
32. How many times $\frac{5}{6}$ in $7\frac{1}{6}$?

Explanation.—Reduce the mixed number to an improper fraction. Ans. $8\frac{3}{5}$.

33. At $\frac{4}{9}$ of a dollar per bushel, how many bushels of potatoes can be bought for \$$13\frac{1}{4}$? Ans. $29\frac{13}{16}$.

34. A gentleman rode in a stage until his fare was $9\frac{4}{19}$ dollars, at 5 cents per mile. How many miles did he ride? Ans. $184\frac{4}{19}$.

Remark.—5 cents equal $\frac{1}{20}$ of a dollar.

35. What number must be multiplied by $\frac{3}{8}$ to give a product of $6\frac{1}{5}$? (See Proof, page 65.)
Ans. $16\frac{8}{15}$.

36. For $73\frac{5}{8}$, how many weeks' board can be obtained at $4\frac{7}{11}$ per week? Ans. $15\frac{359}{408}$.

37. How many cords of wood can be bought for $109\frac{1}{2}$, if 1 cord cost $6\frac{1}{2}$? Ans. $16\frac{11}{13}$.

38. If 7 lb. of tea cost $3\frac{1}{12}$ dollars, what is it per pound? Ans. $\frac{37}{84}$ of a dollar.

39. If a man make 125 barrels in $60\frac{1}{2}$ days, how many barrels does he make per day? Ans. $2\frac{8}{121}$.

40. 5 men purchased $\frac{1}{5}$ of a ship. If they own equal shares, what part of the whole ship did each man purchase? Ans. $\frac{1}{25}$.

41. Bought $3\frac{1}{4}$ yards of cloth for $14\frac{5}{8}$, what did I give per yard? Ans. $4\frac{1}{2}$.

Promiscuous Questions in Vulgar Fractions.

1. What is 4 times the difference between $\frac{21}{20}$ and $\frac{21}{19}$? Ans. $\frac{21}{95}$.

2. What is a 4th part of one third, plus one fourth?
Ans. $\frac{7}{48}$.

3. $\frac{1}{4}+\frac{3}{10}-\frac{1}{6}$, equal what number? Ans. $\frac{23}{60}$.

4. $\frac{2}{7}-\frac{1}{9}\times6\frac{3}{4}\div\frac{2}{5}$, equal what number?
Ans. $2\frac{53}{56}$.

5. $\frac{1}{3}$ of 30 is equal to $\frac{1}{4}$, of what number?

Explanation.—$\frac{1}{4}$ of a number is equal to $\frac{1}{3}$ of the difference between $\frac{1}{4}$ and the whole. If $\frac{1}{4}$ of a number be subtracted from the number, $\frac{3}{4}$ remain, $\frac{1}{4}$ must therefore be $\frac{1}{3}$ of $\frac{3}{4}$. So if we add $\frac{1}{3}$ of a number to the number itself, the $\frac{1}{3}$ will be $\frac{1}{4}$ of the sum. If we take $\frac{1}{3}$ of 30, and add it to 30, the sum 40 will be the answer. But if we divide the one third 10, by $\frac{1}{4}$, the quotient is the answer, for the whole number 10, being brought into quarters by dividing by $\frac{1}{4}$, shows what number 10 is $\frac{1}{4}$ of.

6. $\frac{2}{3}$ *of 18* equal $\frac{3}{4}$ of what number?

$\frac{2}{3}$ of 18=12. Now if we divide 12 by $\frac{1}{4}$, the quotient is 48, which shows that 12 is $\frac{1}{4}$ of 48. But 12 can be $\frac{3}{4}$ of a number only $\frac{1}{3}$ as large as the number of which it is $\frac{1}{4}$, therefore dividing 48 by 3 gives 16 the answer.

7. A merchant bought a quantity of flour, and sold 150 barrels, which was $\frac{1}{5}$ of the whole quantity. How many barrels did he buy? Ans. 750.

8. What is the difference between $\frac{9}{15}$ and 1?

Explanation.—As the denominator represents an unit divided into 15 equal parts, if we take 9 the numerator from the denominator, 6 the remainder, exhibits the difference between 9 and 15; or $\frac{6}{15}$, is the difference between $\frac{9}{15}$ and 1. Ans. $\frac{6}{15}$.

9. A man sold a quantity of grain to 3 men; to A $\frac{1}{3}$; to B $\frac{3}{5}$; and to C 20 bushels, which was the remaining part. How many bushels did he sell to the 3 men?

Explanation.—$\frac{14}{15}$ equal the sum of $\frac{1}{3}$ and $\frac{3}{5}$. Since $\frac{14}{15}$ represent the parts sold to A and B, $\frac{1}{15}$ the remaining part must represent the 20 bushels sold to C. 15 the denominator of the fraction represents the whole quantity, and we may suppose each bushel divided into 15 equal parts, and that A has 5 of those parts, B 9, and C 1. Since 20 is $\frac{1}{15}$ of the whole quantity, multiplying 20 by 15 must give the whole quantity, for the product will contain the multiplicand 15 times. But if we divide 20 by $\frac{1}{15}$, we multiply by 15 and divide by 1, which gives the same number for a quotient, that we had for a product before. Had the numerator of the fraction $\frac{1}{15}$, expressing C's part, been more than 1, C's part must have been a greater part of the whole quantity, and the whole quantity a less quantity:—but in that case, we should have divided by the numerator, which *would have* made the quotient as much less than it *now is*, as the numerator was greater than 1. To prove the work of this question, take $\frac{14}{15}$ of 300, which will denote A and B's part, and subtract it from 300, the remainder will be C's part.

From the three last explanations, we derive the accuracy of the following

RULE,

For finding the whole number, or quantity, when a part or parts of the number or quantity, are represented by fractions, and a part by a simple number.

Add the given fractions; subtract their sum from 1, (or the numerator from the denominator,) and divide the simple number by the fraction found by subtracting, the quotient will be the whole number or quantity required.

10. A man after spending $\frac{3}{8}$ and $\frac{2}{5}$ of his property, had 100 dollars left; what was the value of his property? Ans. \$444$\frac{4}{9}$.

11. A butcher killed an ox, and sold $\frac{1}{2}$ of $\frac{2}{5}$ of it to one man; $\frac{76}{193}$ to a second; and the remainder, which was 300 lb., to a third. What was the whole weight of the ox? Ans. 738$\frac{51}{98}$ lb.

12. What costs a job of work performed by 3 men, if the first man do $\frac{13}{29}$; the 2d. $\frac{13}{39}$; and the third the remainder, for which he receives \$56?

Ans. \$256$\frac{8}{19}$.

13. A man owning $\frac{3}{11}$ of a house, sold $\frac{1}{3}$ of his share to D, and $\frac{1}{4}$ to F. Now, if all the owners except D and F, receive yearly \$208 for their part of the rent, what is the whole rent, and what is D's share of it?

Ans. { Whole rent \$247$\frac{13}{37}$.
D's share \$22$\frac{18}{37}$.

14. A gentleman divided his estate between his 3 sons, as follows: to the 1st he gave $\frac{3}{8}$ of it; to the 2d $\frac{2}{3}$ of the remainder. The difference between the portions of the 1st and 2d, was 500 dollars. What

was the whole estate,—also, what was the 3d son's share?

Ans. { Whole estate $12000.
3d son's share $2500.

15. $\frac{1}{2}$ of a certain farm is wild land; $\frac{2}{7}$ pasturing; $\frac{3}{22}$ mowing; $\frac{1}{17}$ tillage, and the remainder, which is $3\frac{1}{3}$ acres, is orcharding. How much land in said farm. Ans. $174\frac{8}{15}$ A.

16. A certain school house is owned by 14 proprietors, and rented for $24 annually. A and B own each $\frac{1}{15}$ of the house; C $\frac{2}{15}$; D and E each $\frac{1}{24}$; F $\frac{1}{12}$; G $\frac{1}{24}$ and $\frac{1}{2}$ of $\frac{1}{24}$; H $\frac{1}{15}$ and $\frac{1}{4}$ of $\frac{1}{15}$; I $\frac{1}{15}$ and $\frac{1}{2}$ of $\frac{1}{15}$; K $\frac{3}{4}$ of $\frac{1}{15}$; L $\frac{3}{4}$ of $\frac{1}{24}$; M $\frac{23}{480}$; N $\frac{1}{24}$ and $\frac{1}{2}$ of $\frac{1}{15}$; and O $\frac{1}{15}$ and $\frac{3}{4}$ of $\frac{1}{15}$. What does each proprietor receive annually for his share of the rent?

Ans. A and B each $1,60; C $3,20; D and E each $1; F and H each $2; G $1,50; I $2,40; K $1,20; L $0,75; M $1,15; N $1,80; and O $2,80.

◆

DECIMAL FRACTIONS.

The only difference between a vulgar and a decimal fraction, is this, the denominator to a decimal fraction is always 1, with a cipher or ciphers at the right hand of it, while the denominator to a vulgar fraction may be any other number.

Derivation and explanation of Decimal Fractions.

We have already explained how vulgar fractions are derived from division; we shall now illustrate the manner in which decimal fractions are obtained. To do this, we shall make use of several examples.

If 18 dollars be divided equally between 12 men, how many dollars will each man receive?

Operation.

```
12)18(1,5
   12
   ——
    60
    60
    ——
```

It was proved, Contraction 2d, page 45, that annexing a cipher to a number, multiplies that number by 10; 12 can be taken from 18 once, and 6 remains. By annexing a cipher to 6, the remainder, each of the 6 dollars is divided into 10 equal parts, since 60 contains 6 tens. Annexing the cipher does not increase the number of dollars, but denotes the remaining number of dollars divided into parts. By subtracting 12, the number of men, from 60 once, we take 12 tenths of a dollar, which is enough to give each person 1 tenth, and as 12 can be taken 5 times from 60, the whole number of tenths, each man will have 5 tenths besides the 1 dollar.

A comma is set between the 1 and the 5 to denote that the 5 is a fraction. The denominator 10 is understood to stand under the 5, and if written, the fraction would stand thus $\frac{5}{10}$. Had we set 12 the divisor under 6 the remainder, the fraction would have been $\frac{6}{12}$ which equals $\frac{1}{2}$;—but $\frac{5}{10}$ equal $\frac{1}{2}$, for 5 is half of the denominator. The denominator is not written, since any number obtained by adding one cipher to the remainder, will be tenths, and will always be known as such without a denominator written to express it. The answer to the question is therefore $1½, or which is the same thing $1,5.

When any figure stands at the right hand of units' place, and is separated from it by a comma, (which is called a separatrix) it denotes as many tenths of an unit as it contains 1's, whether it be actually obtained from division or not, for it always represents the same thing as though it had been.

If 15 *dollars be equally divided between* 12 *men, how many dollars will each man receive?*

Operation.

```
12)15(1,25
   12
   ---
    300
    24
    ---
     60
     60
     --
```

12 can be taken from 15 once and 3 remains. If we add one cipher to the remainder and divide by 12, there will still be a remainder of 6, we therefore add 2 ciphers which is the same as multiplying the remainder by 100. (See page 45.) Each of the 3 remaining dollars is now divided into 100 parts, and as many times as 12 can be taken from the 300, so many hundredth parts each man will have in addition to the 1 dollar expressed by the first figure in the quotient.

The 25 at the right of the comma, are 25 hundredths and the denominator 100 understood. 25 is $\frac{1}{4}$ of 100, which is equal to $\frac{3}{12}$ the vulgar fraction which would have been formed had we set the divisor under the remainder, therefore the answer is \$$1\frac{1}{4}$, or \$1,25.

If we add three ciphers to the remainder of any question, the remainder will be multiplied by 1000, and each unit of the remainder in that case, would be divided into 1000 equal parts, and that part of the quotient obtained from the remainder after the 3 ciphers are annexed, must be thousandth parts of an unit. In the same manner it might be shown that the more ciphers we annex to the remainder, the less parts of an unit the quotient figures obtained from the remainder, will express. When we annexed one cipher, the fraction was tenths; when we annexed two ciphers, the fraction was hundredths.

Let 21 *be divided by* 12.

Operation.

```
12 ) 21 ( 1,75
     12
     --
      90
      84
      --
       60
       60
       --
```

In this example we annex one cipher to the remainder 9, which reduces it to tenths, and 7 the quotient figure found by dividing the 90 tenths, is 7 tenths. The next remainder 6, is 6 tenths, and by annexing a cipher to it, it divides each of the tenths in the 6 divided into 10 equal parts; one of these small parts must therefore be a tenth of 1 tenth. 5 the right hand figure of the quotient, shows how many of these small parts contained in the 60, there are in the quotient; 5 must therefore be 5 tenths of one of the parts contained in 7,—or in other words, 10 parts as large as those contained in the 5, are equal to 1 of the parts contained in the 7, or equal to 1 tenth of an unit. Annexing the ciphers one at a time is the same thing as though we had annexed them both to 9 and then brought down, the right hand one to 6. In the preceding question, we annexed two ciphers and found the two decimal figures in the quotient to be hundredths; but we have annexed two ciphers in this, therefore, 75 when read together, are hundredths. 9 the first remainder written over the divisor, makes a vulgar fraction equal to $\frac{3}{4}$, showing what part of another unit belongs in the quotient, and 75 being $\frac{3}{4}$ of 100, 75 hundredths denote the same thing.

The whole quotient is $1\frac{75}{100}$, or 1,75, or it is equal to $1\frac{3}{4}$, which may be read one, seven tenths and five hundredths, or one, and seventy-five hundredths, for 7 is the same part of 10 that 70 is of 100.

From what has been said, it appears that every time we annex a cipher to a remainder, the next quotient figure implies parts of an unit, only 1 tenth part as large as those expressed by the figure at the left hand of it. It also appears, that the denominator which is generally understood, will always have as many ciphers in it as the number of ciphers annexed to the remainder, and this number will equal the number of decimal places in the numerator.

The denominator of a decimal fraction equals an unit. If any thing be divided into 10 equal parts, one of the parts is a tenth of the same thing, and any number of those parts less than 10, would be represented by one figure which would stand in the first decimal place; but the denominator of one decimal place is 10, which shows how many of those parts would be required to make a whole. The same might be shown when the denominator is 100, 1000, &c.

An unit is considered as the dividing point between whole numbers and decimals. Whole numbers increase towards the left from unity in a ten fold proportion; decimals decrease towards the right in the same proportion.

From the foregoing explanation we obtain the following general principles.

1. Decimal fractions properly belong to the right hand of whole numbers.

2. The denominator to a decimal fraction is 1 with as many ciphers to the right hand of it, as the numerator has places of figures.

3. 10 in any one place of a decimal, is equal to 1 in the next left hand place.

4. All operations in decimals may be performed as with whole numbers.

5. It is not necessary to write the denominator to a decimal fraction.

Changing Vulgar Fractions to Decimals.

It has been seen in the operations of the questions in the explanation of decimals, that annexing ciphers to a remainder and dividing by the divisor, the quotient figures thus obtained, are decimals expressing the same part of an unit as the vulgar fraction formed by writing the divisor under the remainder. As the numerator of any vulgar fraction represents a remainder, and the denominator the divisor, annexing ciphers to the numerator of a vulgar fraction and dividing by the denominator, the quotient figures, if there be no remainder, will be a decimal expressing the same part of the same unit as the vulgar fraction.

Method of reading Decimal Fractions.

When a decimal fraction stands alone, the numerator is read as a whole number, and the denominator repeated afterwards. ,9 is read nine tenths; ,26 twenty-six hundredths; ,129 one hundred and twenty-nine thousandths, &c. But when a decimal is written in connexion with a whole number, the whole number is first read and then the decimal as before. 2,6 is read two and six tenths; 325,35, three hundred and twenty-five and thirty-five hundredths, &c.

The following table exhibits the names of the decimal places to millionths.

The same numbers as those in the table, are set on the same line at the left hand with the decimals in the form of vulgar fractions.

TABLE.

			Tenths.	Hundredths.	Thousandths.	Ten thousandths.	Hundred thousandths.	Millionths.
$6\frac{5}{10}=$	6	,	5	.	.	.	.	.
$\frac{56}{100}=$		,	5	6	.	.	.	.
$109\frac{126}{1000}=$	109	,	1	2	6	.	.	.
$\frac{4013}{10000}=$		,	4	0	1	3	.	.
$56007\frac{10101}{100000}=$	56007	,	1	0	1	0	1	.
$470\frac{125}{1000000}=$	470	,	0	0	0	1	2	5

Ciphers written at the left hand of decimals, lessen their value, but if written at the right hand do not alter it. ,8 or ,80 is 8 tenths; but ,08 is 8 hundredths, and ,008 is 8 thousandths.*

RULE

For changing Vulgar Fractions to Decimals.

Annex ciphers to the numerator and divide by the denominator.

1. What decimal fraction is equal to $\frac{1}{4}$?
   $100 \div 4 = ,25$ the answer.
2. Change $\frac{3}{4}$ to a decimal. Ans. ,75.
3. Change $\frac{3}{25}$ to a decimal. Ans. ,12.
4. Change $\frac{1}{5}$ to a decimal. Ans. ,2.
5. How many hundredths in $\frac{28}{50}$? Ans. 56.
6. How many thousandths in $\frac{5}{8}$? Ans. 625.
7. A boy gave $\frac{3}{15}$ of a dollar for a pen knife; how many tenths of a dollar did he give? Ans. 2.
8. Change $\frac{100}{250}$ to a decimal. Ans. ,4.
9. Change $\frac{65}{375}$ to a decimal.

Operation.

```
375)650(1733333
    375
    ----
    2750
    2625
    -----
     1250
     1125
     -----
      1250
      1125
      -----
       125
```

---

* The word *decimal* is derived from the Latin word *decem*, which signifies ten.

Decimal Fractions were invented by Regiomontanus, in 1464; but did not come into general use until the last part of the 16th century.

Explanation.—In the operation of this example, after obtaining two places of decimals in the quotient, we find that the same remainder occurs each time, and the same figure is found for a quotient in each succeeding division. This would be the case were we to continue the operation ever so far, for multiplying the same divisor by the same figure each time, we have the same subtrahend which will leave a remainder equal to the preceding one.

After the same remainder is repeated once or twice, the operation may be discontinued and as many figures like the last found in dividing, may be set in the quotient as is thought necessary.

It sometimes happens that different remainders repeat alternately. In examples of this kind, an exact answer in decimals cannot be obtained.

When an exact answer in decimals cannot be found, they are called *Circulating decimals.* If there be a remainder after the operation is discontinued, the sign of addition is annexed, which implies that the answer is not exact.

10. Change $\frac{36}{74}$ to a decimal. Ans. ,4864+.
11. Change $\frac{48}{290}$ to a decimal. Ans. ,16551+.
12. Change $\frac{1}{125}$ to a decimal.

Dec'm.
1·2 5)1 0 0 0(,0 0 8
1 0 0 0

Explanation.—$\frac{1}{125}$ being less than 1 tenth, there will be no decimal in place of tenths. We also observe that after adding two ciphers, the denominator cannot be taken from the numerator, which shows that $\frac{1}{125}$ is less than 1 hundredth. It may be observed generally, that as many ciphers wanting one must be set on the right of the decimal point as we annex ciphers to the numerator before we can divide, which removes the first decimal to its proper place.

13. Change $\frac{3}{376}$ to a decimal. Ans. ,0079+.
14. What decimal equals $\frac{3}{54}$? Ans. ,0555+.
15. How many ten thousandths in $\frac{5}{375}$?
Ans. 133+.

16. 3 farthings equal $\frac{3}{4}$ of a penny. What decimal of a penny is equal 3 farthings? Ans. ,75.

17. 10 pence equal $\frac{10}{12}$ of a shilling. What decimal of a shilling is equal to 10 pence?
Ans. ,8333+.

18. 1 hour is $\frac{1}{24}$ of a day; change it to a decimal of a day. Ans. ,0416+.

19. What decimal part of a day is 11 h. 4 m.?

Explanation.—The 11 h. and 4 m. must first be reduced to a *vulgar* fraction of a day by *Explanation* to question 58th, page 153. Ans. ,461+.

20. If I sell 3 oz. 15 grs. of silver, what decimal part of a pound do I sell? Ans. ,2526+.

21. What decimal part of a mile are 11 inches?
Ans. ,000173+.

22. Reduce 2 A. 3 R. 28 r. 27 ft. to the form of a decimal.

Explanation.—No operation is to be performed upon the 2 acres, the highest denomination. Reduce the less denominations to the decimal of an acre.
Ans. 2 A. and $\frac{925}{1000}$+, or 2,925+A.

23. Change 6 C. 100 ft. 1000 in. to the form of a decimal. Ans. 6,78+C.

24. Reduce 5 yd. 3 qr. 2 na. to the form of a decimal. Ans. 5,875 yd.

25. Reduce 7 E. E. 4 qr. 1 na. to the form of a decimal. Ans. 7,85 E. E.

26. Change $12\frac{6}{13}$ to the form of a decimal.
Ans. 12,461+.

27. Change $100\frac{100}{101}$ to the form of a decimal.
Ans. 100,99+.

28. Change $676\frac{1}{376}$ to the form of a decimal.
Ans. 676,0026+.

ADDITION OF DECIMAL FRACTIONS.

RULE.

Set the numbers under each other so that every figure may stand in a place of its own name, whether decimals or whole numbers. Add as in simple numbers.

The decimal point in the sum must stand directly under those in the given numbers.

1. What is the sum of $16\frac{4}{10}$, $25\frac{9}{10}$, $130\frac{13}{100}$, $275\frac{325}{1000}$, $4000\frac{471}{1000}$?

Operation.

$$\begin{array}{rr}
 & 16,4 \\
 & 25,9 \\
 & 130,13 \\
 & 275,325 \\
 & 4000,471 \\
\hline
\text{Sum,} & 4448,226
\end{array}$$

Illustration.—A figure in any decimal place is so many tenths of 1 in the next left hand place; therefore, when we obtain 10 in adding any column of decimals, 1 added to the next left hand place equals the 10 so obtained, the same as in simple numbers. In the adjoining operation, 6, the sum of the thousandths, not being equal to 1 hundredth, is written in the place of thousandths. 12 hundredths are equal to 1 tenth and 2 hundredths. 1 tenth is added to the column of tenths for 10 of the hundredths, and the two remaining hundredths set in the place of hundredths. 10 tenths equal an unit; in the 22 tenths, we have 2 units and 2 tenths over. The two units added to the column of units, and the 226 thousandths equal all the given decimals.

2. What is the sum of 61,4+113,12+2579,7+190,0793? Ans. 2944,2993.

N. B. When a whole number and a decimal in connexion are expressed in words, the words denoting decimal parts, will generally be found printed in italics, to distinguish them.

This observation applies only to the remaining part of this Section.

3. What is the sum of two hundred and nine, and *sixty-three hundredths*, thirty-five and *five hundredths*, and *fifteen hundredths?* Ans. 244,83.

4. Add together two thousand five hundred and seventeen, and *two hundred* and *forty-one thousandths*, *two hundred* and *nineteen thousandths*, thirty-six thousand, and one thousand and *one hundred* and *one thousandths?* Sum, 39517,561.

5. A man bought three tons and *nine hundredths* of hay at one time; four tons and *sixty-seven hundredths*, at another; fifteen tons and *six hundred* and *seven thousandths*, at another. How much hay did he buy at the three purchases? Ans. 23,367 T.

6. The elevation of a certain mountain above a river near its base, was taken at three observations; one at the base, which was found to be 417 feet and *nine hundred eighteen thousandths*, higher than the river; one upon the side, which was 2000 and *eighteen thousandths* above the base; and one at the summit, which was 3002 and *seven tenths* higher than the place of the second observation. How much higher than the river was the summit of the mountain? Ans. 5420,636 feet.

Note 1.—If no whole numbers are given, the point must be kept at the left hand of decimals, to show that they are decimals; and as many 1's set in the place of units as there are 10's in the column of tenths.

7. What is the sum of ,9+,17+,018+,0037+,90708. Ans. 1,99878.

8. What is the sum of *seven hundred* and *six thousandths*, *two thousand* and *seventy-three hundred-thousandths* and *eight hundred* and *sixty-seventh millionths?* Ans. 727597.

9. Add together *one million*, *two thousand*, *nine hundred* and *fifty-five ten-millionths*, *seventeen thousand* and *ten ten-millionths*, *thirty-five ten-millionths*, and *eight hundred* and *ninety-eight thousandths*.

Ans. 1.

10. What is the weight of 3 bales of cotton; the 1st weighing $319\frac{1}{2}$ lb.; the 2d, $371\frac{1}{4}$ lb; and the 3d, $400\frac{3}{4}$ lb.?

Explanation.—Change the given fractions to decimals. Ans. 1091,5 lb.

11. What is the weight of 4 boxes of raisins weighing as follows: $47\frac{1}{5}$ lb., $54\frac{3}{4}$ lb., $51\frac{3}{5}$ lb., and $55\frac{3}{8}$ lb.?

Ans. 208,925 lb.

12. How much wood in 4 loads which measure as follows: $1\frac{7}{8}$, $1\frac{3}{7}$, $\frac{8}{9}$, and $1\frac{1}{6}$ cords?

Ans. 5,35912+C.

FEDERAL MONEY.

Until 1786, pounds, shillings, pence and farthings, were the currency of the United States.

In 1786 it was enacted by Congress, that the Money of the United States should consist of eagles, dollars, dimes and cents as whole coins,* and that a tenth part of a cent, which is imaginary, should be called a mill. It was also established, that the value of these coins should vary in the same proportion as simple numbers and decimal fractions.

The general name of eagles, dollars, dimes, cents, and mills, is FEDERAL MONEY.

TABLE.

10 mills	*m.*	make	1 cent,	*c.*
10 cents		"	1 dime,	*d.*
10 dimes		"	1 dollar,	*D.* or $.
10 dollars		"	1 eagle,	*E.*

---

* Coin is money stamped with a legal impression.

The *dollar* is considered the unit, and all the less denominations decimal parts of it. The eagle is not named in accounts, or in reading federal money, the whole being expressed in dollars and parts of a dollar.

There are particular coins of the half and quarter eagle, half and quarter dollar, half dime and half cent. The eagle, half eagle, and quarter eagle, are gold coins; the dollar, half dollar, quarter dollar, dime and half dime, are silver coins, and the cent and half cent, copper coins.

1 dime equals 10 cents, 2 dimes 20 cents, 3 dimes 30 cents, &c. 1 cent being a tenth of a dime, is 1 hundredth of a dollar, and 100 cents equal 1 dollar. From this it appears, that we express the same thing, whether we say 2 dollars, 2 dimes, and 5 cents, or 2 dollars and 25 cents. The latter method being the shortest, and the order in which simple numbers are read, is generally used.

$5,645 may be read five dollars, six hundred forty-five mills, or, which is the common method, five dollars, sixty-four cents, five mills. Were the decimal point removed, the whole number would be read, five thousand, six hundred and forty-five mills, for 1 mill being a thousandth part of a dollar, there would be 5 thousand mills in 5 dollars.

Since the parts of a dollar in federal money are decimals of a dollar, all operations in federal money must be performed in the same manner as any mixed number in a decimal form.

13. What is the sum of 25 dollars, 7 dimes, 3 cents, and 4 mills,—37 dollars, 1 dime, 2 cents, and 8 mills,—2 dollars, 5 cents, and 9 mills, and 7 cents, 1 mill?

Operation.

```
  25,734
  37,128
   2,059
    ,071
  ------
Ans. $64,992
```

14. What is the sum of $10,36; $21,09; $103,402; $74,008, and $87,30?

Ans. $296,16.

15. A man sold a quantity of grain for $37 and eight cents; a load of hay for $18 and six cents; a cow for $17, three cents and nine mills, and 5 barrels of cider for $10, five cents and one mill. What was the amount of the whole. Ans. $82,23.

16. If I pay nine dollars for a hat, thirteen dollars and forty-five cents for a pair of pantaloons, and twenty-eight dollars for a watch; what is the cost of the three articles? Ans. $50,45.

17. What is the sum of 92\frac{1}{2}$, 78\frac{5}{8}$, 100\frac{3}{16}$, and 1\frac{6}{18}$? Ans. $272,758+.

18. The daily expenses of a cotton factory are as follows: for labour $100,27; for overseers $20,099; for light, fuel and other articles $46,90. What is the whole expense? Ans. $167,269.

MULTIPLICATION OF DECIMAL FRACTIONS.

Illustration.—It was observed, *Illustration*, page 36, that the product must contain the multiplicand as many times as the multiplier contains an unit; and Illustration, page 145, that when the multiplier is a proper fraction, the product will be such a part of the multiplicand as the fraction is part of an unit; also, in Explanation to question 16th, page 148, that we obtain the product of a whole number multiplied by a proper fraction, by multiplying by the numerator and dividing by the denominator. Every decimal is a proper fraction, therefore multiplying the whole number by the numerator of a decimal fraction, and dividing by the denominator, will give a correct product. Since the denominator to a decimal fraction is always 10, 100, 1000, &c. we have only to multiply by the numerator and point off as many places of figures at the right hand for a decimal or remainder, as there are ciphers in the denominator. (See Contraction 3d and its Illustration, page 71.)

Multiply 641 *by* $\frac{5}{10}$.

Operation 1.

$$\begin{array}{r} 641 \\ 5 \\ \hline \end{array}$$

$$10)3205(320\tfrac{5}{10}$$

$$\begin{array}{r} 30 \\ \hline 20 \\ 20 \\ \hline 5 \end{array}$$

Operation 2.

$$\begin{array}{r} 641 \\ ,5 \\ \hline 320,5 \end{array}$$

In both operations of this example, we obtain the same product; in the first by multiplying by the numerator and dividing by the denominator the same as if the multiplier had been a vulgar fraction; in the 2d, by multiplying by the numerator, and pointing off the right hand figure which is 5 tenths, the denominator being understood. Pointing off the 5 is the same as dividing by 10. 5 tenths equal $\frac{1}{2}$, and by examining the product it will be seen that it is just $\frac{1}{2}$ of the multiplicand; 320 is half of 640, and 5 tenths is half of 1.

Multiply 641 by $\frac{51}{100}$.

Operation.

$$\begin{array}{r} 641 \\ ,51 \\ \hline 6,41 \\ 320,5 \\ \hline 326,91 \end{array}$$

The product of the 1 hundredth can be only the one hundredth part of the multiplicand, which is found by multiplying by the 1 and pointing off the two right hand figures, or dividing it by 100. 6 is the one hundredth part of [illegible] 41 hundredths is the one hundredth part of

41. 50 hundredths equal 5 tenths, so that multiplying by the left hand figure of the multiplier is really multiplying by 5 tenths, which gives 320,5 for a product, the same as in the other example. 326,91, the sum of the two products is the product of both figures.

Multiply 6,41 *by* 51.

Operation.

```
 6 , 4 1
   , 5 1
 -------
   6 4 1
 3 0 5
 -------
3 6 , 9 1
```

In this example we have the same figures but not the same value as in the last. The multiplier is 51 instead of 51 hundredths, and the multiplicand 6 and 41 hundredths instead of 641. According to Illustration, page 43. the product of 41 hundredths multiplied by 51, is the same as the product of 41 multiplied by 51 hundredths, we must therefore have two decimal places as in the last example. It may be shown in another manner, that the two right hand figures will be decimals, for once 1 hundredth is 1 hundredth, and once 4 tenths is 4 tenths, 50 times 1 hundredth are 5 tenths, and the product expresses 5 tenths by setting it under 5 in the multiplier, or in tenth's place. Pointing off the two right hand figures is the same as dividing by 100 the denominator of 41 hundredths in the multiplicand. It will be seen by examining the questions in this division, that when one of the given numbers contains a decimal, by pointing off a number of places equal to the number of places in the given decimal, each part of the decimal product will be brought into the place of its own name by multiplying the same as in simple numbers. The number of ciphers in the denominator of any decimal, is equal to the number of places in the numerator, therefore multiplying as in simple numbers and pointing off as many decimal places as there are places in the given decimal, whether the given decimal belong to the multiplicand or multiplier, will produce a true product. It remains to be seen ho many places must be pointed off for decimals when the multiplicand or multiplier is composed of pure decimals, and the reasons for the same.

Let it be required to multiply $\frac{5}{10}$ by $\frac{5}{10}$.

$\frac{5}{10}\times\frac{5}{10}=\frac{25}{100}$. By multiplying the numerators together for a new numerator, and the denominators for a new denominator, we obtain a correct product, for $\frac{5}{10}$ equal $\frac{1}{2}$, and $\frac{1}{2}$ multiplied by $\frac{1}{2}$, the product is $\frac{1}{4}$, which is equal to $\frac{25}{100}$. But if we set down ,5 and multiply it by ,5, as in the operation, and point off as many decimal places as there are ciphers in the product of the denominators, we have the same answer as before, the denominator being understood. The product of the denominators of any given decimals, will always have as many ciphers as there are decimal places in both of the numerators; to know therefore how many decimal places there will be in any product, we must count the number of decimal places in the multiplicand and multiplier together.

Operation.

,5
,5
____
,2 5

If both the multiplicand and multiplier are mixed numbers, there will be as many decimal places in the product as there are decimal places in both the given numbers, since both decimals will produce as many decimal places in the product as it is composed of.

RULE.

Multiply as in simple numbers, and point as many places of figures at the right hand of the product for decimals, as there are decimal places in the multiplicand and multiplier counted together.

Note 2.—Great care must be taken in setting down the decimal point, for if it be set only one place too far to the right or left, the value of the whole product will be altered ten fold.

1. What is the product of 347,1, multiplied by 6,5? Ans. 2256,15.

2. Multiply 43,01 by 6,04. Prod. 259,7804.

3. What is the product of two hundred and twenty-eight and *sixty-five hundredths*, multiplied by three and *six hundredths?* Ans. 699,669.

4. If a yoke of oxen eat ,45 of 1 cwt. of hay in one day: how much hay will be required to keep them 150 days? Ans. 67,5 cwt.=3 T. 7,5 cwt.

5. If a man build 16,6 rods of fence in one day, how many rods will he build in 11½ days, working at the same rate?

Explanation.—Change the vulgar fraction to a decimal. Ans. 190,9.

6. What will 79 barrels of flour cost, at $4,37 per barrel?

Explanation.—37 cents are 37 hundredths of a dollar, consequently there will be two decimal places in the product, that is, two places of cents. Or if we call the whole price of each barrel 437 cents, which it is, the whole product will be cents, for the 437 cents will be placed in the product once for each barrel. But if we divide the whole number of cents in the product by 100, the number of cents in a dollar, the quotient will be dollars, and the remainder, if any, cents; but the remainder will always be the two right hand figures. From this it appears, that when two places of figures are pointed from the right of any number of cents, the figures to the left of the point, will be dollars, and those to the right of it cents. Ans. $345,23.

7. What cost 79,4 yards of cloth, at $2,36 per yard? Ans. $187,384,

Note 3.—There being one decimal place in the quantity and two in the price of a 1 yard, there will be three decimal places in the product. The three decimal places will be thousandths of a dollar, which are mills. The two left hand decimals are hundredths of a dollar, and the whole decimal may be read three hundred eighty-four mills, or thirty-eight cents and four mills; for dividing the whole number of mills by 10 the number of mills in a cent, the quotient would be cents and the remainder mills, but the two left hand figures would form the quotient and the right hand one the remainder,

8. What is the product of 379765 into seventy-nine thousandths? Ans. 30001,435.

9. What cost 118 gal. of molasses, at 28 cents 4 mills per gal.? Ans. $33.512,

10. What cost 503 yards of tape, at 7 mills per yard? Ans. $3,521.

11. What cost 117 1/7 lb. of butter, at 15 2/7 cents per pound?

Operation.

```
      117,14
       ,1528
      ------
       93712
     ,23428
    5,8570
  11,714
  ----------
$ 17,898992  Ans.
```

Explanation.—In most cases, when a vulgar fraction is reduced to a decimal, it is not necessary to carry the decimal to more than two or three places. In this question we carry the decimal to two places only in the multiplicand. The 2/7 of a centand the cents themselves are decimals of a dollar; the decimals therefore obtained by reducing the 2/7 are decimals of a cent, and the whole multiplier decimals of a dollar.

The decimals to the right of mills, being only thousandths of a mill, may be omitted.

12. What will 26 1/3 barrels of cider come to at a dollar 27 cents and 9 mills per barrel?

Ans. $33,676+.

13. What cost 678 yards of riband, at 13 cents per yard? Ans. $88,14.

14. What is the product of *nine million, seventy-one ten millionths*, multiplied by itself?

Ans. ,81001278005041.

15. What is the product of $6,50, multiplied by *$4,21?* Ans. $27,365.

16. What is the product of 10 dollars, eight cents into 92 cents? Ans. \$9,273+.

Note 4.—The 6 tenths of a mill are omitted.

17. What is the product of 92 cents into 8 cents? Ans. \$0,073+.

18. Multiply 92 cents by 8 mills. Prod. \$0,007+.

19. What will 5675 feet of boards come to at \$9,75 per thousand?

Operation.

```
   5,6 7 5
     9,7 5
   -------
   2 8 3 7 5
 3 9 7 2 5
5 1 0 7 5
-----------
5 5,3 3 1 2 5   Ans. $55,331+.
```

Explanation.—Boards, plank, and joist, are calculated by the thousand. Any number of feet less than 1000 are thousandths of 1000, and the price must be found accordingly. When we read figures denoting any number of feet of boards, plank, or joist, the decimal expression is not generally used. We say five thousand six hundred and seventy-five feet of boards; but in finding the price we must consider 675 feet as thousandths of 1 thousand, for \$9,75 being the price of one thousand feet, is to be taken as often as there are 1000 in the whole number of feet, and a proportional part of it for 675 feet.

20. What is the $\frac{3}{6560476}$ part of 100001? Ans. ,045728+.

21. What must I receive for 10609 feet of boards, at 10 dollars per thousand? Ans. \$106,09.

22. Find what 259 feet of boards will come to, at \$12 per thousand? Ans. \$3,108.

23. What must a ship-carpenter give for 62100 ft. of white oak plank, at \$63½ per thousand? Ans. \$3943,35.

24. What will 900 ft. of plank come to, at 50 dollars and 45 cents per thousand?

Ans. 45 dollars, 40 cents and 5 mills.

25. What will 8 feet of plank come to, at $70 per thousand? Ans. 56 cents.

26. What will 1009 ft. of joist amount to, at $9,25$\frac{1}{2}$ per thousand?

Ans. $933 and 8$\frac{295}{1000}$ mills.

27. How many shillings in, 3 of a pound?

Operation.
2 0
,3
——
6 ,0

Explanation.—By Explanation to question 16th, page 148, it was shown that multiplying a number by the numerator and dividing by the denominator, the quotient would equal the same part of the number as was expressed by the fraction. 20 shillings equal 1 pound, therefore multiplying 20 by 3 the numerator of the fraction, and dividing by 10 the denominator, or pointing off the right hand figure, the quotient will express the same number of shillings as ,3 of a pound. Ans. 6 shillings.

28. How many hours in $\frac{3}{10}$ of a day?

Ans. 7 and ,2 of an hour.

29. In ,26 of a cord of wood, how many solid inches?

Explanation.—First find the number of solid inches in a cord. Ans. 57507,84.

30. If I buy ,09 of a ton of copperas, how many pounds do I buy? Ans. 201,6.

31. How many inches in ,001 of a mile?

Ans. 63,36.

Note 5.—When the value of the decimal is required in several less denominations, reduce the decimal to the lowest denomination mentioned as directed in Rule, page 118, observing to multiply only the decimal each time.

32. What is the value of ,126 of a pound?

Operation.

$$\begin{array}{r} ,126 \\ 20 \\ \hline 2,520 \\ 12 \\ \hline 6,240 \\ 4 \\ \hline ,960 \end{array}$$

Ans. 2 s. 6 d. 0,96 qr.

Explanation.—Multiplying ,126 by 20 and pointing off the 3 right hand figures, gives 2 s. and 52 hundredths of a shilling which is the whole value. By multiplying the decimal of a shilling by 12, gives the value of that part of a shilling in the same manner as in the first part of the operation.

33. What is the value of ,37 of a day ?
Ans. 8 h. 52 m. 48 s.

34. What is the value of ,0101 of a mile ?
Ans. 3 r. 3 ft. 9,936 in.

SUBTRACTION OF DECIMAL FRACTIONS.

RULE.

Place each figure of the subtrahend under one of its own name in the minuend.

Subtract as in whole numbers.

The decimal point must stand directly under those in the given numbers.

Remark.—As decimals increase by 10 the same as whole numbers, we must subtract in the same as in whole numbers. The same illustration is applicable to the subtraction of decimals, which was given for the subtraction of whole numbers. (See page 53.)

1. What is the difference between 659,722 and 437,591? Ans. 222,131.

2. If I pay 67 dollars 47 cents towards a debt of 179 dollars 65 cents, how much of the debt remains unpaid? Ans. $112,18.

3. A handed B a 10 dollar bill to pay a debt of 8 dollars and 9 cents; how much money must B return to A? Ans. $1,91.

4. If I pay all but $37,093 of a debt amounting to $220,10, how much do I pay? Ans. $183,007.

5. What is the difference between 4 cents, 7 mills, and 10 dollars? Ans. $9,953.

6. If 471,972 feet of boards be sold from a pile consisting of three thousand, how many feet of boards will remain? Ans. 2528,028.

7. Subtract nine mills from 9 dollars.
Rem. $8,991.

8. What is the difference between *six thousand, five hundred* and *seventy eight ten thousandths*, and *ninety-three thousandths?* Ans. ,5648.

9, What is the difference between $1\frac{14}{22}$ and $\frac{63}{121}$?
Ans. 1,1157+.

Remark.—It may be proper to observe, that in this, and the two preceding rules, when vulgar fractions are changed to decimals, and the operations performed with the decimals, some of the decimal figures may be different from what they would be were the division in reducing them continued farther. But if three or four places of decimals are found, the difference is so small a part of an unit, that the error is of no consequence in most kinds of business.

10. Subtract $\frac{1}{11}$ from $\frac{2}{9}$.

$$\begin{array}{rl} \frac{2}{9} = ,2\ 2\ 2\ 2\ 2+, & \text{or } ,2\ 2\ 2\ 2\ 2\ 2+ \\ \frac{1}{11} = ,0\ 9\ 0\ 9\ 0+, & \text{or } ,0\ 9\ 0\ 9\ 0\ 9+ \\ \hline ,1\ 3\ 1\ 3\ 2 & \quad ,1\ 3\ 1\ 3\ 1\ 3 \end{array}$$

Explanation.—By carrying the decimal to six places in the second part of the operation, we have 1 in the fifth place; but in the first part of the operation, in which we carry the decimal to 5 places only, the figure in the fifth place is 2.

11. What is the difference between one million and *one millionth?* Ans. 999999,999999.

12. An engineer wishing to ascertain the height of a certain pond above a neighbouring lake, found the summit level of the intervening land to be 24 ft. above the surface of the lake, and 13,3 feet above that of the pond. How much higher was the surface of the pond than that of the lake? Ans. 10,7 ft.

DIVISION OF DECIMAL FRACTIONS.

Illustration.—In dividing decimals, or whole numbers and decimals, the object is to find how many times the divisor can be subtracted from the dividend, or to divide the dividend into several equal parts.

A decimal may be divided by a decimal, and yet the quotient be a whole number.

,5) ,5 (1
5
—

If we divide 5 tenths by 5 tenths, the quotient is an unit, since 5 tenths can be taken from 5 tenths once.

,2) ,8 (4
8
—

So if we divide 8 tenths by 2 tenths, 4, the quotient, is a whole number, for 2 tenths can be subtracted 4 times from 8 tenths.

2 , 3) 4 , 6 (2
4 6
——

Also, the quotint of 4,6, divided by 2,3, is 2 units, since 3 tenths can be taken 2 times from 6 tenths, and 2 units 2 times from 4 units.

```
2 ) 4,8 ( 2,4
    4
    ---
      8
      8
      --
```

But the quotient of 4,8 divided by 2, is a mixed number. The first figure in the quotient is a whole number found by dividing the 4 units in the dividend by 2. The next part of the dividend to be divided is only 8 tenths, and the divisor being a whole number, is equal to 20 tenths; therefore the divisor cannot be taken once from the remaining part of the dividend. If we were to set 20 tenths under the remainder, 8 tenths, the fraction $\frac{8}{20}$ would express the remaining part of the quotient; but $\frac{8}{20}$ equal $\frac{4}{10}$, the same number of tenths which we find by dividing in the common way. The product of the divisor multiplied by the last quotient figure, must never be greater than that part of the dividend which we are dividing. The product of 2 multiplied by 4 tenths, is 8 tenths, which can be taken from the remaining part of the dividend, consequently the right hand figure in the quotient is a fraction. The 4 tenths show what part of the divisor can be taken from 8 tenths.

At $2,5 per yard, how many yards of cloth can be bought for $6,75?

```
2,5 ) 6,7 5 ( 2,7
      5 0
      -----
      1 7 5
      1 7 5
      -----
```

There will be as many yards as 2,5 can be taken times from 6,75. The left hand figure of the quotient is a whole number. The second dividend is 1 and 75 hundredths, a less number than the divisor, and only a part of the divisor can be taken from it, which shows that the next figure in the quotient will be a fraction. Multiplying the divisor by 7 tenths, produces a product of 1 and 75 hundredths, a number just equal to that part of the dividend which is to be divided the second time; 7 tenths must therefore be the last quotient figure. Had there been another decimal place in the dividend, by bringing it to the right of hundredths, the remaining part of the dividend must have been less than that last divided, and the next quotient figure must have been less than tenths. By a similar method of reasoning, it might be shown, that for every additional place in the dividend, there would be an additional place in the quotient.

In the three first examples, the decimal places in the divisor and dividend are equal, and the quotients whole numbers; in the two last, there is one decimal place more in each dividend than in the divisor, and one decimal place in the quotient.

From what has been shown, it appears, that when the decimal places in the dividend and divisor are equal, the quotient may be a whole number, but when the decimal places in the dividend are the greatest, there will be as many decimal places in the quotient as the decimal places in the dividend are more than those in the divisor.

RULE.

Divide as in whole numbers, and point off as many places for decimals in the quotient, as the decimal places in the dividend are more than those in the divisor.

1. How many times can 6,12 be taken from 104,958 ?

Operation.

```
6,12)104,958(17,15 Ans.
     612
     ----
     4375
     4284
     ----
       918
       612
       ----
       3060
       3060
       ----
```

Explanation.—The cipher annexed to the last remainder, does not increase the value of the remainder, but only represents another decimal place in the dividend. Two decimal places are pointed off in the quotient, because the three given decimals, and the cipher annexed to the remainder, make 4 places, two more than the number of decimal places in the divisor.

2. If a yard of cassimere cost $2,31, how many yards can be bought for $50,623?
Ans. 21,91+yards.

3. If I sell 169,8 pounds of butter for $23,26, what do I receive per pound? Ans. $0,136+.

4. A man sells cider to the amount of $65, at $1,95 per barrel; how many barrels does he sell?
Ans. 33,33+.

5. How much sugar, at 12 cents 5 mills per pound, can be bought for $15,50?
Ans. 124.

6. If a dozen of buttons cost 75 cents, what do they cost a piece? Ans. 6 cents and $2\frac{1}{2}$ mills.

7. What is the cost of one bushel of potatoes, if 35 bushels cost $12? Ans. $0,342+.

8. If a labourer receive $4,25 for 6 days' work, what is that per day? Ans. $0,708+.

9. F owes G $15,58, and is to pay him in rye at 67 cents per bushel; how much rye will be required to discharge the debt? Ans. 23,25+bush.

10. When buttons are sold at 9 cents a dozen, what are they a piece?

```
12),090(,0075
     84
     --
      60
      60
      --
```

One button cost 75 ten thousandths of a dollar, or $7\frac{1}{2}$ mills.

Explanation.—The number of cents the 12 buttons cost being less than 12, the price of one button is less than a cent. A cipher is set before the 9 cents to show that the 9 cents are hundredths. By annexing a cipher to 9, the dividend becomes 90 mills, and the first figure obtained in the quotient, is mills; the second tenths of a mill, found by annexing a cipher to the remaining 6 mills.

13. What is the quotient of 6 divided by 125?
Ans. ,048.

14. What is the quotient of 26, divided by 2,6?

```
2,6 ) 26,0 ( 10
      26
      ——
        0
```

Explanation.—20 contains 2 ten times, and 6 contains 6 tenths, ten times, but dividing in the common way, we get only the figure 1 in the quotient, which is in reality 10. A cipher is written in the first decimal place, which being brought down, places a cipher in the quotient, and the 1 is thus removed into tens place. We may consider the divisor 26 tenths; then annexing a cipher to the dividend, that may be called 260 tenths, and the question would be, how many times can 26 tenths be subtracted from 260 tenths.

When the decimal places in the dividend are not equal to those in the divisor, they must be made equal by annexing ciphers, otherwise the quotient figures will not express their true value.

15. If a person spend $100 in one year, what is that per day? Ans. $0,273+.

16. The President of the United States receives a salary of $25000 a year; how much is that per day?
Ans. $68.493+.

17. A general wishes to distribute $12000 among his army, consisting of 3250 men; giving an equal share to each; what will each man receive?
Ans. $3,693+.

18. At a dollar and seventy five cents a barrel for cider, how much is that per gallon?
Ans. 5 cents and 5+mills.

19. Divide 1,656 by 1656.

1 6 5 6) 1,6 5 6 (1
1 6 5 6
———

Explanation.—The 1 in the dividend is the 1 thousandth part of the 1 in the divisor, and the 656 thousandths are the 1 thousandth part of the 656, and only a 1 thousandth part of the divisor can be taken from the dividend. The figure 1 in the quotient is the 1 thousandth part of an unit, and to make it express that we must prefix two ciphers. Ans. ,001.

Note.—From the last example it appears that when the number of places in the quotient are not so many as what the number of decimal places in the dividend are more than those in the divisor, they must be made equal by prefixing ciphers.

20. If 112 lb. of iron cost $7,25 what is the cost of one pound? Ans. $,064+.

21. If I pay $13¼ for my stage fare 220 miles, what do I give per mile?
Ans. 6 cents and 1+mills.

22. Divide 1 hundredth by 10. Quot. ,001.

23. Divide 2 millionths by 1 million.
Quot. ,000000000002.

Promiscuous questions in Vulgar and Decimal Fractions.

1. What is the sum of $\frac{1}{3}$, $\frac{7}{9}$, ,65 and ,078?
Ans. 1,839+.

2. What is the difference between $\frac{11}{15}$ and ,0179?
Ans. ,7154+

3. What is the product of $\frac{1}{5}$ of a shilling into ,2 of a pound? Ans. ,002£.

4. What is the quotient of ,11 divided by $\frac{13}{99}$?
Ans. ,83+.

5. A gentleman gave a lot of land containing 126 acres to two sons,—giving one 9 acres more than the other; how many acres did each son receive?

Explanation.—There are several methods by which this question may be performed. If we subtract 9 from the whole number of acres, and divide the remainder by 2, the quotient would be his share, who has the least number of acres, and the 9 added to the quotient would give the other's share. Or, if the whole number of acres be divided by 2, and half of 9 be substracted from the quotient, the remainder would be the less number, and half of 9 added to half the given number of acres would be the greater number. Ans. One 67,5 and the other 58,5.

6. Divide 100 into two such parts, that one shall be 12½ greater than the other.
Ans. 56,25 and 43,75.

7. If water run into a cistern at the rate of 15 gallons an hour, and run out at the rate of 7¼ gallons in the same time, how much time will be required to fill the cistern, if it contain 500 gallons?
Ans. 64,51 hours.

8. For a certain pile of wood a gentleman agreed to give $34,67. After measuring it, he found that it contained 5 C. 96 ft.; what did the wood cost him a cord? Ans. $6,029+.

9. At 3½ cents a foot, what will 12 plank come to which measure 56 ft. 9 in. each? Ans. $23,835.

10. A merchant bought a hogshead of molasses for $23, and sold it for $6 more than he gave; at what did he sell it for per gallon? Ans. $0,46+.

11. A man buys a chest of tea weighing 40 lb. for $35; at what must he sell it for per pound to gain 10 dollars on the whole? Ans. $1,12½.

12. If two men start at the same time from different places, and travel towards each other,—one at the rate of 4 miles an hour, and the other ⅔ of a mile less; in how many hours, and at what distance from each place, will they meet, if the distance between the two places be 66 miles?

Ans. They will meet in 9 hours—36 miles from one place and 30 from the other.

SECTION VIII.

PER CENT.*

DEFINITIONS.

1. *Per Cent.* implies that calculations are made by the hundred.

2. *Rate Per Cent.* specifies the particular number of units calculated on 100.

3. *Interest* is a sum of money given by a debtor to a creditor as a reward for the use of money.

4. *Principal* is the money on which interest is calculated.

5. *Amount* is the sum of the principal and interest.

6. *Commission* is money paid to an agent for transacting business for another.

7. *Discount* is so much per cent. to be deducted from any given sum of money.

8. *Insurance* is a contract by which one person agrees to pay another the value of property when lost at sea, destroyed by fire, or in any other manner mentioned in the contract, for a certain per cent. paid the insurer by the insured.†

9. *Premium* is the sum of money paid the insurer by the insured.

10. *Duties* are certain sums of money paid to government by importers of goods, wares, merchandize, &c.

11. *Par* is a word denoting that money, or stock, is worth its nominal value.

12. *Stock* is a general term for the capitals of trading companies, or a fund established by law.

6 per cent. is 6 cents calculated for 100 cents, 6 dollars for 100 dollars, 6 shillings for 100 shillings,

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* *Per* signifies *by*. Cent. is from the Latin word centum meaning 1 hundred.

† Insurance is generally made by corporate companies.

6 pounds for 100 pounds, &c. 5 per cent. is 5 cents calculated for 100 cents, 5 dollars for 100 dollars, 5 pence for 100 pence, &c.

Interest, discount, commission, premiums, and generally, duties, are calculated by the hundred.

To find the commission and premium on any sum of money;—also to find the interest on any sum for 1 year.

RULE.

Multiply the given sum by the rate per cent.

Note 1.—When interest is mentioned at so much per cent. it is always understood for one year unless otherwise stated in the question.

1. What is the interest of $126, at 6 per cent.?[*]

Operation.

```
  1 2 6
   ,0 6
 ------
$7,5 6 Ans.
```

Illustration.—At 6 per cent. 6 cents is the interest for 100 cents, or 1 dollar, for one year, and multiplying the number of dollars by 6 cents, the product contains 6 cents once for each dollar in the multiplicand, (See Proof and Illustration, page 43,) consequently, the product must be the interest of the number of dollars multiplied. In the operation of question 1st, we multiply by 6 hundredths of a dollar, the product must therefore be divided by the denominator 100, which is effected by pointing off the two right hand figures, (See Illustration, page 189,) 7 must therefore be dollars and 56, hundredths of a dollar, that is, cents. 756 cents contain 6 cents as many times as there are dollars in 126, and is the interest for 1 year.

2. What is the interest of $560, at 6 per cent.?
Ans. $33,60.

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[*] The rate per cent. by which interest may be taken is established by the State Legislatures, and in most of the States is 6 per cent. In New-York it is 7 per cent.

3. Compute the interest on $1000, at 6 per cent.?
Ans. $60,00.

4. What is the interest of $105, at 7 per cent.?
Ans. $7,35.

5. What is the interest of 67 dollars and 67 cents, at 6 per cent.?

```
  6 7,6 7
     ,0 6
---------
$4,0 6 0 2
```

Explanation.—The interest of 1 cent can be only 100th part of the interest of a dollar. By pointing off four decimal places instead of two, we lessen the value of each figure 100 times.

6. What is the interest of $720,60, at 5 per cent.?
Ans. $36,03.

7. What is the interest of $129,35, at 7 per cent.?
Ans. $9,054+.

8. What would be the commission on the sale of goods to the amount of $4820, at 3 per cent.?
Ans. $144,60.

9. If an agent sell goods to the amount of $1250 on a commission of 4½ per cent. what must he receive?

Explanation—4½ per cent. equal ,045.

```
2)1 2 5  0               1 2 5 0
     , 0 4½               , 0 4 5
  ----------             ---------
  5 0 0  0                6 2 5 0
    6 2  5             4 9 0 0
  ----------           -----------
Ans. $5 6,2  5         $5 6,2 5 0 Ans.
```

10. A merchant in Boston paid 1000 dollars for a quantity of hats. After sending them to Philadelphia he employed a merchant of that city as agent, who sold them for $1250,50, and was to receive a commission of 2¾ per cent. on the original cost. What did he receive?
Ans. $27,50.

11. What is the premium on $12756 insured, at 4 per cent.? Ans. $510,24.

12. An insurance company in New-York insured a ship's cargo, valued at $11027,69, for 4½ per cent. Soon after the vessel sailed, a violent storm induced the owner to apply to the same company for insurance upon the vessel, which was valued at $13005, and insured at 17⅕ per cent. What premium did the owner pay for insurance on vessel and cargo?

Ans. $2733,10+.

13. What is the premium on a ship's cargo, invoiced at $15000, at 2½ per cent.?

Ans. $375.

14. What is the premium for insuring a house against fire, valued at $1129, at 1 per cent.?

Ans. $11,29.

Note 2.—Insurance on buildings is made for a specified time. The premiums are calculated on an annual per cent.

15. A gentleman obtained insurance upon his buildings, valued at $2000, for ½ per cent., and upon his furniture, valued at $1200,18, for ⅜ per cent.; what was the sum of both premiums?

Ans. $14,50+.

16. What premium must be given for insuring a stock of goods against fire, at $1\frac{2}{7}$ per cent., if the goods are valued at $7010,95?

Ans. $90,14+.

17. The machinery in a cotton factory was valued at $45'000. On ¾ of this sum insurance was made for 1½ per cent., and on the remainder for 1⅙ per cent. What was the premium on the whole?

Ans. $637,50.

Duties are computed on some kinds of goods at certain rates per cent. ad valorem; on other kinds, at a certain rate for a particular weight or measure, as a ton, pound, gallon, or yard, &c. [Ad valorem implies that a certain per cent. on the cost is to be added before the duties are computed.]

If 10 per cent. of the actual cost is to be added to the invoice* before the duties are computed, we may increase the invoice or the rate by $\frac{1}{10}$ of itself for we shall obtain the same result, whether we augment the multiplicand or multiplier, if we increase either by the same part of itself. 5 is $\frac{1}{10}$ part of 50, and 55×10=550; but $\frac{1}{10}$ of 10 is 1, and 50×11=550; the same product.

18. What is the duty on a quantity of wine, the invoice of which is $2600, at 40 per cent., if the invoice be raised 10 per cent.?

Explanation.—$\frac{1}{10}$ part of 2600, is 260, and 260 added to 2600, makes $2860, the sum on which the duties are to be reckoned. $2860×40 per cent. gives $1144, the answer or duty on 2860, at 40 per cent.

10 per cent. on 40 per cent. is 4 per cent., which added to the given rate makes 44 per cent. Multiplying $2600 by 44 per cent. gives $1144 the same amount of duty as before. The duty on the wine is actually 44 per cent.

19. What is the duty on 25 chests of tea, which cost $16 each, at 50 per cent., if the invoice be increased 20 per cent.

Explanation.—20 per cent. of 50, the rate of duty is 10 per cent. The actual per cent. on the cost of the tea is 60 per cent. Ans. $240.

20. What is the whole cost of a lot of books, the invoice of which is $743,66, at 15 per cent., raising the invoice 10 per cent.? Ans. $866,36+.

21. By what per cent. must the actual cost of goods be multiplied, when the rate of duty is 43 per cent., and the invoice is to be increased 20 per cent.?

$$\begin{array}{r} ,43 \\ ,20 \\ \hline ,0860 \end{array} \qquad \begin{array}{r} ,43 \\ ,086 \\ \hline ,516 \end{array} \text{ Ans.}$$

---

* Catalogue of the several articles which make up a vessel's cargo. It is here used to denote the cost of the cargo.

22. By what per cent. must the cost of goods be multiplied to obtain the duty, when the rate of duty is 31½ per cent., and 10 per cent. is to be added to the invoice ? Ans. ,3465.

23. A merchant in Philadelphia imported a quantity of saltpetre, for which he paid $500 exclusive of freight, which was $206. What was the whole cost of the saltpetre after paying a duty of 8¼, or ,0825 per cent. ?

Explanation.—The cost of freight must be added before the duties are cast.

Ans. 764,24½.

24. Bought a lot of books, the price of which was $127,09 ; but for cash a discount of 30 per cent. was made. What was the real cost of the books.

Explanation.—30 cents for each dollar of the price is to be subtracted. Ans. $88,963.

25. A bought of B 28 bushels of peas at $1,07 per bushel ; but proving not so good as was expected, B agreed to make a discount of 7 per cent. ; what did A pay ? Ans. $27,86+.

26. What is the discount on $762,999, at ¼ per cent. ? Ans. $1,907+.

27. Bought a quantity of bibles, for which the seller charged $1,10 a piece, but made a discount of ⅓ for money down ; what was paid a piece ?

Ans. $0,73+.

28. If 537 pounds of ginger are sold for $53,70, what would it be a pound if 2½ per cent. discount were made ? Ans. $0,097+.

Note 3.—The value of stock is variable. When 100 dollars of stock are worth 100 dollars in money, the stock is said to be at par ; when worth more, to be above par ; when less, below par.

If 100 dollars in stock are worth 105 dollars in money, the value of the stock is 105 per cent., but if worth only 95 dollars, the value is 95 per cent. The shortest method of finding the real value of any given amount of stock, is to multiply the given stock by its value per cent.

29. What is the value of $12450 United States' bank stock, at 112 per cent.?

Explanation.—112 per cent. is 1 dollar and 12 cents, that is, 1 dollar of stock is worth 1 dollar and 12 cents in money. 1 dollar being an unit does not alter the value of the multiplicand, as may be seen in the third product, which is placed by the common method of multiplying to the left of the decimal point.

```
     1 2 4 5 0
         1,1 2
     ---------
     2 4 9,0 0
   1 2 4 5,0
 1 2 4 6 0
 -------------
$ 1 3 9 4 4,0 0   Ans.
```

The answer must contain the given stock, and 12 cents for each dollar in the given stock. This is obtained by multiplying by the 1 dollar and 12 cents, since 12 cents are placed in the product for each unit in the multiplicand. Or, 12 cents are $\frac{12}{100}$ of a dollar, and according to Illus. page 189, and Expl. page 193, multiplying by $\frac{12}{100}$, the product will be the same part of the multiplicand that $\frac{12}{100}$ are of a dollar.

30. What is the value of $101 of stock, at 90 per cent. ¿

Explanation.—The real value of the stock can be only such a part of $101 that 90 cents are of a dollar; and the product will express that part by multiplying by 90 cents the numerator, and pointing off two places at the right hand for decimals, or cents, which is dividing by 100 the denominator.

Ans. $90,90.

31. What is the value of $7625 United States bank stock, at 104$\frac{3}{4}$ per cent.?

Ans. $7987,187+.

32. If I invest $6000 in a common stock, and the stock fall $2\frac{1}{4}$ per cent., what would my share be worth? Ans. $5865.

33. A bought 3 shares of stock for $625,50, after the purchase the stock fell $3\frac{1}{3}$ per cent.; what was A's loss? Ans. $20,85.

Note 4.—When interest is required for 2, 3, 4, &c. years, multiplying the interest for one year by the number of years must give it, for the product will contain the multiplicand, which is the interest for one year, as many times as there are years in the multiplier.

34. What is the interest of $127 for 2 years, at 5 per cent.? Ans. $12,70.

35. What is the interest of $1703,78 for 3 years, at 6 per cent.? Ans. $306,68+.

36. What is the interest of $21 for 4 years, at 7 per cent.? Ans. $5,88.

37. What is the amount of $700,50 for 6 years at 6 per cent.?

See definition 5, Sect. 8. Ans. $952,68.

38. What is the interest of 91 cents and 3 mills for 5 years, at 8 per cent.? Ans. $0,365+.

39. What is the amount of two hundred thirty-two dollars and seven cents for 3 years, at $5\frac{1}{4}$ per cent.?

Ans. Two hundred sixty-eight dollars, sixty-two cents and $1\frac{25}{1000}$ mills.

40. What is the amount of $1019,17 for 3 years, at 5 per cent.? Ans. $1172,045+.

Note 5.—When interest is cast at 6 per cent., it is 6 per cent. for 12 months, which is 1 per cent. for 2 months; the per cent. for 1 month must therefore be $\frac{1}{2}$, or $\frac{5}{10}$ per cent.; for 3 months, $1\frac{1}{2}$, or 1, 5 per cent.; for 4 months, 2 per cent., &c. It thus appears that when the rate is 6 per cent. for 12 months, the rate of any time will be half of the number of months.

Remark.—½, ¼, &c. per cent., used in reference to Federal money, is ½, ¼, &c. of 1 cent or $\frac{1}{100}$ of a dollar; therefore when we multiply by a part of one per cent., if the place of tenths and hundredths are not supplied with ciphers, there must be two more decimal places in the product than those in the multiplier and multiplicand.

41. What is the interest of $720 for 8 months, at 6 per cent.?

$$\begin{array}{r} 720 \\ ,04 \\ \hline \$28,80 \end{array} \quad \text{Ans.}$$

42. What is the interest of $1005,50 for 11 months, at 6 per cent.?

$$\begin{array}{r} 1005,50 \\ ,055 \\ \hline 502750 \\ 502750 \\ \hline \$55,30250 \end{array} \quad \text{Ans.}$$

The per cent. for 10 *months is* 5, *and for* 1 *month*, 5 *tenths of* 1 *per cent.*

43. What is the interest of $325,54 for 13 months, at 6 per cent.? Ans. $21,16+.

44. What is the interest for 74 cents for 1 year and 5 months, at 6 per cent.? Ans. $0,062+.

45. What is the amount of $100,10 for 14 months, at 6 per cent.? Ans. $10,807.

*Note.*6—$\frac{1}{12}$ of 365 is 30, 4$\frac{2}{12}$ days. As all the Calendar months in a year, except February, have either 30 or 31 days, in casting interest on small sums, it will be sufficiently accurate to call each month 30 days. 6 days are $\frac{1}{10}$ of 60, since 60 contains ten 6's; therefore dividing any number of days less than 60 by 6, the quotient will be a decimal of 1 per cent., or the per cent. for the days.

The most convenient method of finding the per cent. for any number of months and days, is to add 30 days for the odd month (if any) to the given number of days; divide the sum by 6, and annex the quotient to half the even number of months, which will give the per cent. for the given months and days.

46. What is the interest for 69 dollars and 27 cents for 9 months and 24 days, at 6 per cent.?

```
   6 9,2 7
    ,0 4 9
  ---------
  6 2 3 4 3
2 7 7 0 8
-----------
3,3 9 4 2 3
```

Explanation.—The per cent. for 8 months is 4, and for the odd month and 24 days, or 54 days, is $\frac{9}{10}$ of 1 per cent.; or in other words, the per cent. for the whole time is 4 cents and 9 mills.

Ans. $3,394+.

The student will find little difficulty in ascertaining the per cent. for any months and days, when he reflects that the per cent. for two months is 1 per cent., and for every 6 days 1 mill.

47. What is the interest of $117 for 15 months and 15 days, at 6 percent.? Ans. $9,067+.

48. A note of $121 was given January 3, 1825, and paid January 15, 1826. The interest was cast at 6 per cent.; what was due on the note at the time of settlement?

For finding the time, see Explanation, page 107.

Ans. $128,502.

49. D gave a note for 200 dollars and 98 cents, July 7, 1823, and paid it July 1, 1825; what was the interest on said note at 6 per cent.?

Ans. $23,916+.

50. A merchant bought goods to the amount of $975, January 31, 1824, for which he gave his note on interest after 90 days. What was there due on said note, August 1, 1826, allowing interest at 6 per cent.? Ans. $1106,625.

51. What is the interest of $133 for 8 months and 15 days, at 5 per cent.?

```
        1 3 3
      , 0 4 2 5
     ----------
        6 6 5
      2 6 6
    5 3 2
    -----------
6 ) 5,6 5 2 5
     , 9 4 2 0
  -------------
  $4,7 1 0 5 Ans.
```

Explanation.—$5,6525 is the interest at 6 per cent., and ⅙ part of the interest at 6 per cent., which is 94 cents and 2 mills, is the interest at 1 per cent., therefore the difference between $5,6525 and $0,9420 is the interest at 5 per cent. Had the interest been 4 per cent., twice the interest at 1 per cent., or ⅓ at 6 per cent., should have been subtracted. When the interest is more than 6 per cent., take corresponding parts and add them.

52. What is the interest of 26 cents for 7 months and 29 days, at 7 per cent.?

Explanation.—When there is a remainder in taking ⅙ of the days, it will not be necessary to carry the decimal farther than hundredths of one per cent.? Ans. $0,012+.

53. A gave a note April 27, 1824, for $100, at 8 per cent. interest, with the following condition,—that if the note were paid within 1½ years, the rate should be reduced to 6 per cent. The note was paid January 30, 1826; what was due on the note at that time? Ans. $114,066+.

54. What is the interest of $38 for 8 months, at 6 per cent.?

	3 8	
	,0 6	
6 mo.=$\frac{1}{2}$ of a year	2 , 2 8	interest for 1 year.
2 mo.=$\frac{1}{6}$ of a year, or $\frac{1}{3}$ of 6 months	1 , 1 4	interest for 6 months.
	3 8	interest for 2 months.
	$1 , 5 2	interest for 8 months.

Explanation.—The interest for one year is $2,28. 6 months equal $\frac{1}{2}$, and 2 months $\frac{1}{6}$ of a year, therefore $\frac{1}{2}$ and $\frac{1}{6}$ of $2,28 added together, is the interest for 8 months.

55. What is the interest of $70,09 for 11 months 15 days, at 4 per cent.?

	70,09	
	,04	
6 mo.=$\frac{1}{2}$ of a year	2,8036	interest for 1 year.
4 mo.=$\frac{1}{3}$ of a year	1,4018	interest for 6 months.
1 mo.=$\frac{1}{4}$ of 4 mo.	,9345	interest for 4 months.
15 days=$\frac{1}{2}$ of 1 mo.	,2336	interest for 1 month.
	,1168	interest for 15 days.
	$2,6867	in. for 11 mo. and 15 d.

Explanation.—$0,9345 is the interest for 4 months, and $0,2336 interest for 1 month. 15 days equal $\frac{1}{2}$ of a month, therefore $\frac{1}{2}$ of $0,2336 is the interest for 15 days.

56. What is the interest of $62,25 for 3 years, 11 months and 21 days?

	62,25	
	,06	
	3,7350	
	3	
6 mo.$=\frac{1}{6}$ of 3 years	11,2050	interest for 3 years.
3 mo.$=\frac{1}{2}$ of 6 mo.	1,8675	interest for 11 mo.
1 mo.$=\frac{1}{3}$ of 3 mo.	,9337	
1 mo.$=1$ month	,3112	
15 days$=\frac{1}{2}$ of 1 mo.	,3112	
5 days$=\frac{1}{3}$ of 15 days	,1556	interest for 21 days.
1 day$=\frac{1}{5}$ of 5 days	,0518	
	,0103	
	$14,8463	in. for 3 y. 11 mo. 21d.

Explanation.—When the months are not a single part of a year, that is, when the numerator is more than one, the most ready course is to take a number of months which will form a single part, as in the last question. Observe the same method when the days are not a single part of a month.

57. What is the interest of $1,07 for 2 years, 7 months and 9 days, at $4\frac{1}{2}$ per cent.?

Ans. $0,107+.

58. W gave his note June 15, 1823, for 1200 dollars, at 5 per cent. interest. The note was payable in 15 months from date; but W wishing to defer the payment, agreed to add $2\frac{1}{2}$ per cent. to the rate of interest from the time the note was due until payment was made. What was the amount of the note May 2, 1826, when payment was made?

Ans. $1421,75.

59. What is the interest on a note of $17,18, at 5 per cent., dated January 2, 1826, and paid February 25th, in the same year? Ans. 0,126+.

As the exact interest is not found by the two preceding methods, we shall now examine a method by which the interest may be found with a greater degree of accuracy.

60. What is the interest of $25,249 from January 1, 1825, to June 15, of the same year, at 5 per cent.?

```
      2 5 , 2 4 9
            , 0 5
      -----------
      1 , 2 6 2 4 5
            , 4 5 4
      -------------
        5 0 4 9 8 0
      6 3 1 2 2 5
    5 0 4 9 8 0
  -----------------
  , 5 7 3 1 5 2 3 0
```

Ans. $0,573+.

Explanation.—The time is 5 months and 14 days, (not reckoning the day of settlement.) If we multiply 30 by the number of months, and add 14, the whole number of days thus found is 164, which is one day short of the true time. In most cases by adding 1 day for 2 months, the time will be found within 1 day,—always within 2 days,—and frequently the exact time. By adding two days in this question, the time is 1 day too much, but the sum is so small that the error is but a little more than 2 mills. $1,26245 is the interest for 1 year, the interest for the given time must be $\frac{166}{365}$ of the same, therefore multiplying the interest for 1 year by 166, and dividing by 365, the quotient will be the interest required; but if we change $\frac{166}{365}$ to a decimal, we find ,454 which is nearly equal to $\frac{166}{365}$, consequently multiplying the interest for one year by ,454, and pointing off three more places of decimals, the product must be the same part of the interest for one year, that the multiplier is part of an unit or 1 year. (See Illustration, page 148.)

61. What is the interest of $18 for 10 months, 23 days, at 4 per cent.?

The time is $\frac{328}{365}$ of a year. Ans. $0,646+.

62. What is the interest on a note of $96 for 2 years, 8 months, at 5 per cent.? Ans. $12,806.

63. What is the interest of 10 cents for 1 Y. 7 mo. 28 d. at 7 per cent.? Ans. $0,011+.

64. What is the interest of $500 for 25 days, at 6 per cent.? Ans. $2,04.

65. At 12 per cent., what will be the interest of $40 for 1 month and 20 days? Ans. $0,657+.

66. What is the amount of $9 from February 29, 1824, to December 23 of the same year, at 5½ per cent. interest? Ans. $9,403+.

Note 7.—It will not be necessary generally, to carry the decimal of the days farther than to 3 places; but if greater accuracy be required, they may be carried to 4 or 5 places.

In the following 4 questions the exact time is found.

67. What is the interest on a note of $470,06, dated November 19, 1823, and paid June 16, 1826, at 6 per cent.?

Explanation.—It is 2 years from November 19, 1823, to November 19, 1825. From November 19, 1825, to June 16, 1826, there are 209 days—counting 11 in Nov., 31 in Dec., 31 in Jan., 28 in Feb., 31 in March, 30 in April, 31 in May, and 16 in June. ,572 of a year equal 209 days, and 2,572 years is the multiplier, by which the interest for one year must be multiplied.

Ans. $72,539+.

68. What is the interest of a note of $1100,50 dated October 21, 1824, and paid Sept. 20, 1826, at 4½ per cent.? Ans. $94,835+.

69. A gave his note of 725 dollars, August 21, 1824, to be on interest after 60 days, at 6 per cent. What was due on the note, Sept. 21, 1826?

Ans. $806,157+.

70. What is the interest of $29 from May 3, 1824, to July 4, 1825, at 5⅔ per cent.?

Ans. $1,922+.

Remark.—We have explained 3 methods of finding interest, when the time comprises a part of a year.

The three following rules contain directions for finding interest, derived from those explanations.

RULE 1.

When the rate is 6 per cent. for 1 year, find the per cent. for the given time by annexing $\frac{1}{6}$ of the days contained in the odd month, (if any,) and the given days to half the even number of months, and multiply the principal by the same.

Note 8.—When the per cent. is any other than 6, add or subtract $\frac{1}{6}$ for every one the given per cent. is more or less than 6. If the per cent. be 5, subtract $\frac{1}{6}$; if 4, $\frac{2}{6}$; if 7, add $\frac{1}{6}$, &c.

RULE 2.

Multiply the interest for one year by the number of years, and add such a part of the interest for one year as the months and days are parts of a year.

Note 9.—Parts of the interest for any number of years, may be taken for the interest of months and days, by observing what part of the number of years the months and days are.

RULE 3.

Reduce all the time less than a year to the decimal of a year, and annex the decimal to the number of years; then multiply the interest for one year by the mixed number thus found, or by the decimal, if there be no years.

How to find the interest for days only, estimating the year at 360 days.

Estimating the year at 360 days agreeably to Note 6, Section 8, the per cent. for 6 days is 1 tenth of 1 per cent.,—that is, 1 mill, when the given rate is 6; therefore, multiplying the principal by as many mills as there are 6's in the number of days, gives the interest.

71. What is the interest of 22 dollars for 24 days, at 6 per cent.? Ans. 8 cents, 8 mills.

72. What is the interest of 57 dollars 24 cents for 27 days, at 6 per cent.?

Ans. 25 cents 7+mills.

73. What is the interest of $2,10 for 25 days, at 6 per cent.? Ans. $0,008+.

74. What is the interest of 10 dollars for 59 days, at 5 per cent.? Ans. 8 cents and 1+mill.

To find the interest on notes when partial payments are made.

RULE.

Cast the interest on the whole note from the date to the time a settlement is made, and add the interest to the principal. Then compute the interest on each endorsement from the time it was made to the time of settlement, and add the interest to the endorsement. Add the amount of all the endorsements together, and subtract the sum from the amount of the whole note, the remainder will show what is still due.*

75. A note of $137, was dated July 7, 1825, and paid June 14, 1826. January 2, 1826, an endorsement of $38 was made; what was due when the note was paid, interest at 6 per cent.?

Ans. $105,668+.

Illustration.—The design of the preceding rule is, that the partial payment or endorsement is to pay so much of the principal, and to stop the interest on an equal sum. By computing interest on the whole note after a part of it is paid, the interest computed on the part paid, is not due; but by computing the interest upon an equal sum for the remaining time, and subtracting the amount from the amount of the whole note, just balances the excess of interest found on the note, and also lessens the note by a sum equal to the payment.

---

* The interest in the examples under this rule is computed by *Rule 3d*, page 221.

76. A note of $546,60, was dated December 25, 1823, on which were the following endorsements: January 3, 1825, $110, and August 4, 1825, $20,50; what was due July 31, 1826, allowing interest at 6 per cent.? Ans. $489,737.

77. A merchant bought goods to the amount of a thousand dollars, and gave his note, dated Jan. 1, 1824, on interest after 90 days. Six months after the date of the note, he paid $560; and five months after the first payment, he paid 406 dollars. What was due August 23, 1826, computing interest at 6 per cent.? Ans. $63,525+.

78. A bought a horse for seventy-five dollars, and paid two-fifths at the time. The remainder he paid in fifteen dollar payments, and the interest, on the whole, at four and three-fourths per cent., at the last payment. The time intervening between the payments was 30 days. What was the last payment?

Ans. $15,351+.

79. A note of $1200,67, at 7 per cent. interest, was dated November 7, 1823, on which the following partial payments were made:

April 6, 1824,	$371,22,
January 3, 1825,	$197,00,
November 14, 1825,	$469,35.

What was due July 4, 1826, and what part was interest?

Ans. { The sum due, $286,405+,
of which $123,305+ was interest.

Remark.—The methods of computing interest on notes when partial payments are made, vary in different sections of the country. The student, however, will find little difficulty in computing interest by any method, after he has acquired a thorough knowledge of the general principles.

COMPOUND INTEREST.

Interest is compounded when the interest on the principal for one year, or the time agreed upon, is added to the principal, and the interest cast on the sum for another period of time.

Compound interest is not sanctioned by law, but is frequently computed by the consent of the parties concerned. There is no difference between the methods of casting compound interest and simple, except adding the interest at the end of a specified time, and making the sum a new principal for another period of time. If no particular agreement is made, the interest is generally added to the principal at the end of each year.

80. What is the amount of a hundred and ten dollars, for four years, at six per cent., compound interest.

Operation.

```
              110
              ,06
             ----
             6,60 interest for the 1st year.
          110,
          -------
          116,60 principal for the 2d year.
              ,06
          -------
          6,9960 interest for the 2d year.
       116,60
       ----------
       123,596 principal for the 3d year.
           ,06
       ----------
       7,41576 interest for the 3d year.
    123,596
    -----------
    131,011 principal for the 4th year.
        ,06
    -----------
    7,86066 interest for the 4th year.
  131,011
  -------------
$138,871 amount for 4 years.
```

The decimals of a mill are omitted.

81. What is the compound interest of $409,53, for 3 years, at 5 per cent.?

The interest will equal the difference between the whole amount and the given sum.

Ans. $64,552+.

82. What is the amount of $117,06 for 2 Y. 5 mo. 19 days, at 6 per cent. compound interest?

Ans. $135,22.

83. What was the amount of a note of $1005, Oct. 1, 1826, bearing date Dec. 12, 1823, at 5½ per cent. compound interest? Ans. $1167,973+.

PROMISCUOUS QUESTIONS IN PER CENT.

1. Estimating the year at 360 days, what is the per cent. for each of the following periods of time, when the per cent. for a year is 6; 7 days, 17 days, 28 days, 1 month and 5 days, 2 months and 19 days, 7 months and 16 days? (See Note 6, Sect. 8.)

Answers,—,00116+,—,00283+,—,00466+, —,00583+,—,01316+,—,03766+.

2. At 6 per cent. for one year, what is the per cent. for 34 days, estimating the year at 365 days?

34 *days equal* $\frac{34}{365}$ *of a year. The per cent., therefore, for* 34 *days is* $\frac{34}{365}$ *of* 6 *per cent., which is* $\frac{204}{365}$ *of* 1 *per cent., or changed to a decimal*—,00558+.

3. If the per cent. for 365 days be 5, what would be the per cent. from Jan. 1, to June 17, of the same year? Ans. ,02287+.

4. A man bought a quantity of cotton for 800 dollars, and sold it so as to gain 9 per cent. For how much was the cotton sold?

Explanation.—He gained 9 cents on each dollar of the purchase money, consequently, multiplying 800 by 9 cents, the product will be the gain.

Ans. $872.

5. A buys a quantity of rice for $179,56; for what must he sell it to gain 11 per cent.?

Ans. $199,311+.

6. A merchant buys 117 hats for $325; at what must he sell the hats a piece to gain 12 per cent.?

325 times 12 cents added to 325 dollars, will be the amount for which he sells the whole.

Ans. $3,111+.

7. If I pay 16 cents a pound for 137 lb. of butter, what must I sell the whole for, to gain 15 per cent.?

Ans. $25,208.

8. H bought a quantity of cambrick for 131 dollars, and sold it for $157; what did he gain per cent.?

Explanation.—The difference between what he gave and what he received, is the whole gain, and the whole gain divided by the number of dollars he gave, will show what was gained on each dollar, which will be the per cent. 157—131=26 the whole gain; and the per cent. is the quotient of 26 divided by 121.

The per cent. may be expressed by a fraction thus, $\frac{26}{131}$ of a dollar, which reduced to a decimal equals $0,198+the answer.

9. A merchant bought a quantity of salt on board of a vessel for 41 cents per bushel, and paid 2 cents per bushel for truckage. He sold the salt for 57½ cents per bushel; what did he gain per cent.?

Ans. ,3372+.

10. A man bought an ox for 39 dollars, and sold it for 38½; what did he lose per cent.?

When the price given is dollars, and the difference between the giving and selling price is cents, the price given must be reduced to cents by adding two ciphers. The loss per cent. in the last question is $\frac{50}{3900}$, or. ,0128+*the answer.*

Note 10.—The difference between the cost of any quantity and the price for which the same quantity is sold, divided by the cost, will give the per cent. gained or lost, whether the price for which the quantity is bought and sold, be dollars or cents.

If 2 dollars be divided by 10 dollars, the quotient is 2 dimes, or 20 cents,—that is, 20 cents, or 20 per cent. on each dollar in the divisor. The quotient of 2 cents divided by 10 cents, is 2 mills,—that is, 2 mills for each cent in the divisor; but if we multiply the divisor, 10 cents, by 100, and 2 cents the dividend by 100, the quotient will be the same as in the other example; ,2 mills, therefore, gained or lost on 1 cent is the same per cent., as 20 cents gained or lost on 1 dollar.

11. P buys 8000 pounds of tobacco for $850, and pays cash for $\frac{1}{2}$. For the remainder he gives a note payable in 6 months, on interest at 6 per cent. after 30 days. For what must he sell the tobacco per pound to gain 25 per cent.?

Ans. 13 cents and 4+mills.

12. A merchant bought 3 T. 4 cwt. of iron, June 21, 1825, for 5 dollars 25 cents per hundred, and sold it Dec. 6, following, for 6$\frac{3}{7}$ dollars per cwt. He paid 7$\frac{1}{2}$ per cent. for the use of the purchase money; what did he gain? Ans. $63,872+.

13. If I buy cloth at 26 cents per yard, what must I sell it for to gain 10 per cent., if I pay 6 per cent. for the use of the purchase money?

Ans. $0,301+.

14. D bought a quantity of flour for $4,50 per barrel, but it proving bad, he is willing to lose 15 per cent.; for what will he sell it per barrel?

Ans. $3,825.

15. A man sold a piece of cloth for 25 cents a yard, and by so doing he lost 20 per cent.; what did it cost him per yard?

Explanation.—He sold it for $\frac{80}{100}$ of what it cost him; or, in other words, the price for which he sold is in the same degree less than the cost, that the numerator 80 is less than the denominator 100.

Were we to take $\frac{80}{100}$ of the cost the result would be the price for which he sold, but if we invert the terms of the fraction, thus, $\frac{100}{80}$, and multiply 25, the price for which it was sold, by $\frac{100}{80}$, the result will be a number in the same degree larger than 25, that 100 is larger than 80.

Ans. $0,3125.

16. If a man lose 26 per cent. by selling 16 bushels of apples for 5 dollars, what did the apples cost him per bushel? Ans. $0,422.

17. A merchant sells a quantity of paper for 20 dollars, and loses 16 per cent.; what ought he to sell it for to gain the same per cent.?

Ans. $27,618+.

18. In what time will 527 dollars gain 158 dollars 10 cents, at 6 per cent. interest?

There will be as many years as the interest of 527 dollars for one year can be taken times from $158,10.

Ans. 5 years.

19. G paid H 169 pounds of currents, at 11 cents a pound, for the use of $107,38 a certain time, at 5 per cent. per annum; how long did G keep H's money?

Operation.

```
5,369)18,590(3 years.
      16107
      -----
       2483
         12
      -----
5,369)29,796(5 months.
      26845
      -----
       2951
         30
      -----
5,369)88,530(16 202/413 days.
      5369
      -----
      34840
      32214
      -----
       2626
```

Explanation.—The price of the currants is $18,590, and the interest of $107,38 for one year is $5,369. The price of the currants divided by one year's interest, gives the time, which is 3 Y. and $\frac{2483}{5369}$ of another year. Multiplying the numerator of the fraction by 12, the number of months in a year, and dividing by the denominator, gives the value of the fraction in months, which is $5\frac{2951}{5369}$. To obtain the value of $\frac{2951}{5369}$ of a month in days, we multiply the numerator by 30, and divide by the denominator, and find $16\frac{2626}{5369}$, or $16\frac{202}{413}$ days. Therefore, the answer is, that G had the use of the money 3 Y. 5 mo. $16\frac{202}{418}$ d. (See Explanation to Question 16, page 148; also, Explanation to Question 32, page 197.)

20. A mechanick was to give an apprentice $110 when he should be 21 years old, which took place March 15, 1820. At that time a new contract was made, by which the apprentice was to have the loan of $500 until the interest at 5 per cent. should balance the $110. At what time was the apprentice to repay the $500? Ans. August 9, 1824.

21. B gave a note of $400, dated March 31, 1824; and January 1, 1826, when he paid the same, it amounted to $433,25. At what rate per cent. did he pay interest?

Divide the interest for 1 year by the principal.

The time is $1\frac{3}{4}$ years, and $\frac{4}{7}$ of the whole interest is the interest for 1 year? Ans. $4\frac{3}{4}$ per cent.

22. If I pay $42,0875 for the loan of $259 for $3\frac{1}{4}$ years, at what rate do I pay interest?

Ans. 5 per cent.

23. A has corn worth 73 cents per bushel cash, but in barter he lets B have it for 85 cents, and takes tea which B is selling for 95 cents cash; at what per pound must A receive the tea in barter?*

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* When one commodity is exchanged for another, the trade is called *barter*.

The object in this question is to find a number in the same degree larger than 95, *that* 85 *is larger than* 83. *This will be obtained by taking* $\frac{85}{73}$ *of* 95.

Ans. $1,106+:

24. If hops, which cost 30 cents per pound, are sold for 34 cents, at what price should pepper be sold, which cost 31 cents per pound, to gain the same per cent.? Ans. $0,351$\frac{1}{3}$ per pound.

25. A shoe-merchant sells 300 pairs of shoes worth $1,35 ready money, for $450, and takes his pay in brick worth $5 per thousand. How many brick does he in equity receive? Ans. 81 thousand.

26. In 1810, the number of inhabitants in New-York was 959,049, and in 1820, 1,372,812. What was the gain per cent. in population from 1810 to 1820? Ans. 43,1+.

27. Two men put their money together for the purpose of trading; one put in $500; the other $750. They gained 49 per cent. on their stock; what was the actual gain of each?

Ans. One gained $245; the other 367\frac{1}{2}$.

28. Three men united their capitals in trade; viz. A, $3500; B, $3050; and C, $5000. After continuing in business a certain time, A found his share of the common stock to be worth $4750. What per cent. did they gain; also, what was the value of B and C's shares?

Ans. They gained 35$\frac{5}{7}$ per cent. B's share $4139,28+, and C's share $6785,71+.

29. D and E after trading together, found their capital to be worth $3500, and that they had gained 40 per cent. on their original stock. At that time E's share was $2100; what did they put in respectively?

Explanation.—Were we to subtract the whole gain from $3500, the remainder would be the stock with which they commenced trade. Now, if we divide what they gained by 40 cents, the quotient will express the number of dollars put into trade, since the gain on each dollar was 40 cents. Also, if we divide the number of dollars put into trade by one dollar, the quotient will be the same number as the dividend. But if we reduce $3500 to cents, by annexing two ciphers, and divide by 140 cents, every time we take 140 cents from the 350000 cents, we take 1 dollar of the original stock, and 40 cents the gain on that dollar, the quotient will therefore be the same number that we should have obtained had we divided the original stock by 1 dollar, and the gain by 40 cents. This is the shortest method to find the original stock, when the per cent. is given; and the gain and original stock are expressed in the same number. E owned $\frac{2100}{3500}$, or $\frac{3}{5}$ of the stock.

Ans. E $1500, and D $1000.

30. A man by trading several years gained $1427, which was $31\frac{1}{2}$ per cent. on the money with which he commenced trade. What was his original capital?

Ans. $$4530\frac{10}{63}$.

31. C after trading awhile with D, took $625, which was $\frac{3}{7}$ of the whole gain; how much money did D unite with C, allowing they gained 29 per cent.?

Ans. $$2873\frac{49}{87}$.

32. W with a capital of $7000, proposes to take X as a partner, whose capital is only $800, on condition that X shall be entitled to $\frac{1}{2}$ of the whole gain by paying W interest, at 5 per cent., on such part of W's stock as will make their stocks equal. If their united stocks at the end of one year amount to $8700, what will X gain in the trade?

Ans. $295.

* * *

Remark.—Most of the questions which occur in business, are solved by the principles already illustrated. We shall here insert a number of questions, many of which will vary in some respects from those which have been given, but all of them may readily be worked by the student who has obtained a knowledge of the principles explained in the preceding sections, without particular rules. When a question is given, the first object of the student should be, to study well the value of the question; to ascertain what is required, and by what means that result may be obtained.

One reason why scholars are so often perplexed in working questions in arithmetick, is because they go to work before they distinctly understand the precise object to be obtained.

QUESTIONS TO BE SOLVED BY THE PRINCIPLES ALREADY ILLUSTRATED.

1. What number equals 5 times the sum of 69. and 127 and 1000? Ans. 5980.

2. What number equals $\frac{1}{9}$ of the difference between 2767 and 28? Ans. $301\frac{1}{3}$.

3. What number must be multiplied by 13 to make a product 1 less than 183? Ans. 14.

4. What number must be divided by 21, that the quotient may be 35? Ans. 735.

5. A man bought 35 bushels of barley, and sold the whole for $15. He made $2,75 in the trade; what did he give per bushel? Ans. 35 cents.

6. A merchant bought 76 gallons of brandy, at 91 cents per gallon, and sold the whole for $85,16. What did he gain on one gallon?

Ans. $21\frac{1}{19}$ cents.

7. A merchant bought 5 pieces of cloth, each measuring 31 yards, for $26,35. What did he sell it for per yard to make $5 on the whole?

Ans. $20\frac{7}{31}$ cents.

8. A gentleman bought a cask of wine containing 46 gallons for 76 cents per gallon. At what must he sell it per gallon to gain $6,27, if three gallons are lost by the cask's leaking? Ans. $0,958+.

9. If a man walk two miles in half an hour, how far can he walk in 6 days, when the days are $10\frac{1}{2}$ hours long? Ans. 252.

10. One pound sterling equals \$4,44. How many dollars in 35 pounds sterling?

Ans. \$155,40.

11. If I buy 120 gallons of rum for \$75, how much water must be added to it, that I may sell it for 70 cents per gallon, and gain \$12 in the sale of it?

The quotient of 8700 *divided by* 70 *cents, will express the whole number of gallons after the water is added.* Ans. $4\frac{2}{7}$ gal.

12. 60 is $\frac{7}{8}$ of a certain number; what is that number?

See Illustration page 166, and Explanations in promiscuous questions in vulgar fractions.

Ans. $68\frac{4}{7}$.

13. 22 is $\frac{11}{13}$ of a certain number; what is $\frac{1}{3}$ of that number? Ans. $8\frac{2}{3}$.

14. A merchant bought $\frac{1}{5}$ of $\frac{2}{7}$ of a ship for \$467,28; what was the ship worth?

Ans. \$8177,40.

15. If $\frac{3}{13}$ of an acre produce 28 bushels of potatoes, how many bushels will 4 acres produce, at the same rate? Ans. $485\frac{1}{3}$.

16. A gentleman buys 2 lots of land, containing 110 acres each, for \$1500. What is the value of an acre of land in each lot, if an acre in one lot be worth $1\frac{1}{2}$ acres in the other?

Ans. \$5,45$\frac{5}{11}$ and \$818$\frac{2}{11}$.

17. If 2 yards of cloth cost 5 dollars, what will 17 yards cost?

First find what 1 yard is worth; then multiply the value of one yard by the number of yards.

Ans. \$42,50.

18. If 100 dollars gain 3 dollars in 6 months, how much will \$450 gain in the same time?

Ans. \$13,50.

19. If 3 men build 8 rods of wall in 1 day, how many rods will 7 men build? Ans. $18\frac{2}{3}$.

20. If 3 men build 8 rods of wall in 1 day, how many rods will 7 men build in 7 days, working at the same rate? Ans. $130\frac{2}{3}$.

21. A gave $269 dollars for 23 pieces of linen. What did one piece cost him;—also what did $4\frac{1}{2}$ pieces cost?

Ans. { One piece cost $11,69$\frac{13}{23}$.
{ $4\frac{1}{2}$ pieces cost $52,63$\frac{1}{23}$.

22. If 20 acres produce 35 tons, 4 cwt. and 23 lbs. of hay, neat, what does 3 acres produce?

Ans. 5 T. 5 cwt. 2 qr. 13,45 lbs.

23. If 2 barrels of flour would last a family of 12 persons 3 months, how long would the same quantity last if 5 were added to the family?

Ans. $2\frac{2}{17}$.

24. D and E together can do a piece of work in 18 days. D can do the same in 30 days; in what time can E perform it?

Explanation.—If D can do in 30 days the labour that would require the united labour of D and E 18 days, D can perform in 12 days the same work which would require E 18 days to perform,—consequently dividing 30 days, the term required for D to do the work, by $\frac{12}{18}$, or $\frac{2}{3}$, will give the time in which E could perform the same.

Ans. 45 days.

25. A man employs 3 men to do a certain piece of work,—one of whom can do it in 40 days; one, in 36 days,—and the other in 25 days. In what time will they perform it together?

Ans. $11\frac{2}{9}$ days.

26. B bought 200 lbs. of alum for $16; what part of 16 dollars did he give for 1 pound?

Ans. $\frac{1}{200}$.

27. If I pay $9,50 for 100 pounds of sugar, what part of the $9,50 do I give for $3\frac{1}{2}$ pounds?

Ans. $\frac{7}{200}$.

28. If I give $\frac{1}{5}$ of a dollar for $\frac{3}{8}$ of a bushel of corn, how many bushels can I buy at that rate for 1 dollar?

Divide $\frac{3}{8}$ *by* $\frac{1}{5}$. Ans. $1\frac{7}{8}$.

29. A farmer borrows 18 bushels of rye, when it is worth 68 cents per bushel; how many bushels must he return to pay the value of what he borrowed, when it is worth 80 cents per bushel?

The price of 18 *bushels at* 68 *cents per bushel, divided by* 80 *cents, will give the answer.*

Ans. $15\frac{3}{10}$.

30. H borrowed 56 pounds of wool, when it was worth 45 cents per pound, and agreed to return the same value of wool at the expiration of two years, with interest at 6 per cent. At the time of payment, wool was worth but 36 cents a pound; how much wool was H bound to return?

Ans. 78 lb. $6\frac{2}{5}$ oz.

31. A and B are 4 miles apart, and commence travelling towards each other at the same time. A takes 5 steps of 2 ft. 7 in. each in the same time that B is travelling 10 ft. What distance will each travel before they meet?

Explanation.—A travels 155 inches in the same time that B is travelling 120; therefore one will travel $\frac{120}{275}$, and the other $\frac{155}{275}$ of the whole distance.

Ans. { A travels 2 m. 2 fur. 1 r. 7 ft. 6 in.
B " 1 m. 5 fur. 38 r. 9 ft.

32. If two men start at the same time from 2 places $1\frac{1}{2}$ miles apart, and travel in the same direction,—the forward one at the rate of $2\frac{1}{3}$ miles an hour, and the other 3,—how long before the latter will overtake the former?

He has $1\frac{1}{2}$ *miles to gain, and he gains* $\frac{2}{3}$ *of a mile in an hour, therefore* $1\frac{1}{2}$ *divided by* $\frac{2}{3}$ *will be the time.*

Ans. $2\frac{1}{4}$ hours.

33. L and O commenced trade at the same time. L's capital was $1000, on which he gained 20 per

cent. annually; O's $1200, on which he gained 12 per cent. How long a time did it require for L's money to equal O's?

Ans. 3 Y. 6 mo. 25 d.

34. If I owe $100 to be paid in one year without interest, what sum at the present time will pay the debt?

Explanation.—This question may be performed in a similar manner to question 29th, page 230.

As money is always worth a certain per cent., when money is due at some future day without interest, it is generally agreed that a sum, which put to interest at the present time, would amount to the whole debt in the given time, at the lawful rate of interest; ought to pay the debt. The sum which will pay the debt at the present time, is called the *present worth.* $100 will amount to $106 in 1 year at 6 per cent., therefore $\frac{100}{106}$ of $100 will amount to $100 in one year at 6 per cent. Ans. 94,33$\frac{51}{53}$.

35. What is the present worth of $450 due in 2 Y. 6 m. discounting at the rate of 7 per cent.?

Ans. $382,97$\frac{41}{47}$.

36. N owes $2765; his property is worth but $2165; what can he pay P, to whom he owes $350.

$\frac{2165}{2765}$ *of* 35 *will be the answer.*

Ans. 274\frac{4}{79}$.

37. R is worth only $\frac{11}{13}$ of what he owes; how much can he pay S, to whom he owes $1009?

Ans. 853\frac{10}{13}$.

38. If a man can pay his creditors only 76 cents on a dollar, how much can he pay on a debt of $567.80?

Ans. $431,528.

39. If a man's property be worth $1000, and he owes $1200, how many cents can he pay on one dollar?

Ans. 83$\frac{1}{3}$.

40. A gentleman wishing to purchase 100 yards of *silk*, calls upon A, who offers him that quantity for

$98 cash. Being desirous to make a better bargain, the purchaser requests C to make him his best offer. C informs him that he will let him have silk of the same quality of A's, for 1 dollar per yard, and will wait 6 months for his pay without interest. Which is the best offer, and by how much, allowing money to be worth 4 per cent. interest?

Ans. A's by $3\frac{47}{51}$ cents.

41. If $18 will pay 3½ week's board, how long can board be obtained, at the same rate, for $144?

Ans. 28 weeks.

42. How many lengths of a chain 4 rods long, will there be in measuring the 5 sides of a field, which are each 81½ rods? Ans. $101\frac{9}{16}$.

43. A merchant exchanges 75 yards of shirting, at 13 cents per yard, for 65 pounds of butter; what is the butter a pound? Ans. $0,15.

44. What will 2 tons of hemp come to at 6 mills per pound? Ans. $26,88.

45. If a retailer give $66,20 for a cask of rum containing 120 gallons, what must he sell it for a gill, to gain $49 on the whole? Ans. 3 cents.

46. A merchant bought 5 pieces of muslin, each 31⅓ yards, for 2 s. 6 d. per yard, and sold it so as to gain a sum equal to ⅕ of the cost. What did he receive per yard? Ans. 3 s.

47. If 1 C. and 29 solid feet of wood cost 8 dollars, what will 8 cords cost?

Ans. $52,17+.

48. A market-man sells a merchant 300 lb. of pork, at 5½ cents a pound; 16½ bush. of rye, at 68 cents per bush.; and 87¾ lb. of honey, at 16 cents a pound. He receives in part pay, 10 bush. of salt, at 70 cents per bush.; 50 lb. of cotton, at 17½ cents a pound; and 39 gal. of molasses, at 47 cents per gal. For the remainder the merchant agrees to pay cash, if the market-man will take a 5 dollar bill of uncurrent money, at 15 per cent. discount; to which he consents. How much current money is the market-man to receive? Ans. $3,43.

49. How much wood in a pile 96 ft. 6 in. long, 4 feet wide, if $\frac{1}{4}$ of the length is 3 ft. 3 in. high, and $\frac{1}{3}$, 3 ft. 10 in. and the remaining part 4 ft.?

Ans. 11 C. 42 ft. 2 in. 2″.

50. At 9 s. 7 d. for 3 lb. of coffee, what will $\frac{2}{3}$ of 2 lb. come to? Ans. 4 s. $3\frac{1}{9}$ d.

51. At $13\frac{1}{3}$ cents for $\frac{1}{4}$ of a pound of cinnamon, what will 4 lb. 5 oz. come to?

Ans. $2,30.

52. A has $\frac{1}{3}$ of a yard of broadcloth for which he gave at the rate of $$8\frac{1}{2}$ per yard. He gives the broadcloth and 50 cents for $1\frac{1}{4}$ yards of cassimere. What did the cassimere cost him per yard?

Ans. $$2,66\frac{2}{3}$.

53. A gentleman was 6 days travelling from Boston to Washington city. He paid $6\frac{1}{4}$ cents per mile for stage fare, and $1,75 a day for other expenses. His whole expense was $37,75; what is the distance between the two places?

Ans. 436 miles.

54. A gentleman left his estate to be divided between his widow, two sons and a daughter as follows: the widow's share was to equal the elder son's share plus $525; the elder son's to equal the younger son's plus the daughter's; the younger son's to equal $\frac{1}{3}$ of $\frac{3}{5}$ of the whole estate; and the daughter's share to equal $\frac{3}{7}$ of the younger son's share. What was the share of each, and the amount of the whole estate?

See Rule, page 175.

Ans.
Widow's share, $1575.
Elder son's " $1050.
Younger son's " $735.
Daughter's " $315.
Whole estate, $3675.

55. At 6 o'clock the minute and hour hands of a clock point in opposite directions; how long before they will first point in the same direction.

Ans. $32\frac{8}{11}$ minutes.

SECTION IX.

INVOLUTION.

DEFINITIONS.

A Root is a number from which any power of that number may be obtained by being multiplied into itself a certain number of times.

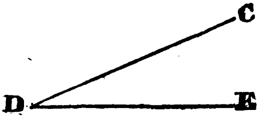
A Power is the last product of a root multiplied into itself a given number of times. [If 5 be multiplied by itself, 5 is the root and 25 the 2d power.]

An Index is a small figure or figures set to the right, and a little above the root, to denote what power is required. [Thus, 21^2 signifies the 2d power of 21.]

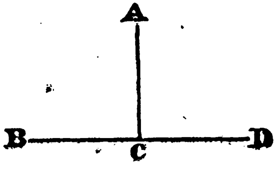
A Line is length without breadth or thickness.

A Right, or Straight Line is a line which does not change its direction, or it is the shortest distance between two points; as A B.

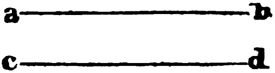
An Angle is the opening of two lines when they meet in the same point, or it is the space included between them when they proceed from the same point, as C D E. When an angle is read, the letter standing at the point where the lines meet, is read in the middle, as C D E, or E D C.



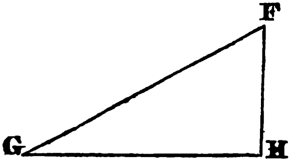
A Perpendicular Line is one which forms a right angle with another line. [If the angle A C B be equal to the angle A C D, the line A C is perpendicular to the line B D, and the angles A C B and A C D are right angles.]



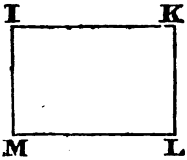
Parallel Lines have the same perpendicular distance between them in every part, as *a b* and *c d*.



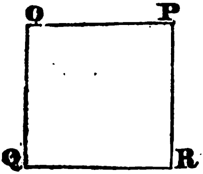
A Triangle is a figure having three sides. If one of the angles in a triangle be a right angle, the triangle is called a *right angled triangle.* F G H is a triangle.



A Parallelogram is a figure of four sides, having the opposite sides parallel. If I K be parallel to M L, and K L to I M, the figure I K L M is a parallelogram. When the angles in a parallelogram are right ones, it is called a *rectangle.*



A Square is a figure having 4 equal sides, and 4 right angles. If O P be equal to, or as long as each of the other sides, and the angles Q O P, O P R, P R Q, and R Q O, be right angles, the figure O P R Q is a *square.*



Involution is the method of finding a required power from a given root.

Remark.—The 2d power of a number is generally called the *square*, and the third power, the *cube* of that number.

RULE,

For finding any power of a given root.

Multiply the given root into itself as many times as the index has units wanting one, the last product will be the power required.

Note.—The reason for multiplying once less than the number of units in the index, is because the root itself, or given number, is called the first power, and the first product must be the 2d, and the 2d product, the third power,[*] &c.

1. What is the square, or 2d power, of 8?
Here $8\times8=64$ the square of 8.
2. What is the cube, or 3d power, of 8?
$8\times8=64$, and $64\times8=512$ the answer.
3. The square of 15 is how much? Ans. 225.
4. What number is equal to 16^3?
Ans. 4096.
5. The 20^4 is how much? Ans. 160000.
6. What is the square of 1? Ans. 1.
7. The 3^3 is how much? Ans. 27,
8. What number is equal to 7^2? Ans. 49.
9. 9^3 is equal to what number? Ans. 729.
10. The 5^5 is how much? Ans. 3125.
11. What is the sum of 1^4, 2^3, 3^2? Ans. 18.
12. Required the sum of 2^5, 4^4, 5^3, and 50^2.
Ans. 2913.
13. What is the sum of $1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2, 8^2, 9^2$? Ans. 285.
14. What is $1^3+2^3+3^3+4^3+5^3+6^3+7^3+8^3+9^3$? Ans. 2025.
15. What is the difference between 37^3 and 45^4?
Ans. 4'049'972.
16. What is the product of 13^2 multiplied by 13^3?
Ans. 371'293.
17. What is the 5th power of 13?
Ans. 371'293.
18. What is the product of the 2^3 into the 2^8?
Ans. 2048.

---

[*] The power of a number is so called because it is what the root *is just capable of producing*, when multiplied into itself.

19. What is the 11th power of 2?

Ans. 2048.

In the 16th question, we multipled the third power of 13 by the square of 13; in the 17th question, we raised 13 to the 5th power, and found the same number as when we multiplied the 3d power of 13 by the 2d power of 13. In the 18th question we multiplied the 3d power of 2 by the 8th power of 2, and obtained the same number, as when we raised 2 to the 11th power in the 19th question. By examining these four questions, it will be seen that multiplying two powers of the same number together, we shall obtain that power of the same number, or root, whose index is equal the sum of the indic[illegible] of those powers multiplied into themselves. In the 16th question, the sum of the indices is 5, and in the 17th question, the given index is 5;—in both these questions we obtain the 5th power of 13. The sum of the indices in the 18th question, is 11, a number equal to the given index in the 19th, and in both questions we find the 11th power of 2.

This principle will hold true in all cases, for multiplying one power by another, we increase the power multiplied as much faster than we should were we to multiply by the root itself, as the multiplier is greater than the root. If the 4th power of 4 be required, we may obtain it by multiplying 4 three times into itself, as in the first operation below, or by multiplying the 2d power of 4, which is 16, by 16, as in the 2d operation.

Operation 1.	Operation 2.
4	16
4	16
16	96
4	16
64	256
4	
256	

64 contains 16 four times, and the product of 64 multiplied by 4, must contain 16 sixteen times,—but the product of 16 multiplied by 16, is made up of 16 sixteen times.

It is frequently more convenient to find the powers of numbers by the method pursued in the 2d operation.

20. What is the 6th power of 279 ?
Ans. 471'655'843'734'321.

21. Required the difference between the 2d and 4th powers of 27. Ans. 530'712.

22. What is the quotient of the 4th power of 12, divided by the cube of 4 ? Ans. 324.

23. $11^3 \div 11^2$ equals what number ?
Ans. 11.

24. What is the square of 25,6 ? Ans. 655,36.

25. What is the cube of 7 ? Ans. 343.

26. What is the square of $14\frac{1}{2}$? Ans. 210,25.

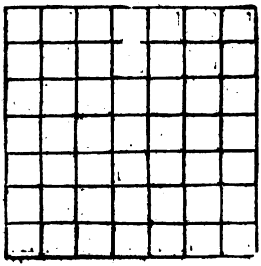
Note.—When the exact value of a vulgar fraction can be found in a decimal, the better way will generally be, to change it before finding the power.

27. What is the cube of $2\frac{3}{5}$? Ans. 17,576.

28. What is the cube of ,001 ?
Ans. ,000000001.

29. How many square rods in a garden in the form of a square, if the length of one side be 7 rods ?
Ans. 49.

FIG. A.



It was observed, page 96, that multiplying the length of a surface by the breadth, gives the contents,—but as the length and breadth of a square are equal, multiplying the length of one side by itself, that is, squaring the given side, or finding the 2d power, will give the area, or contents.

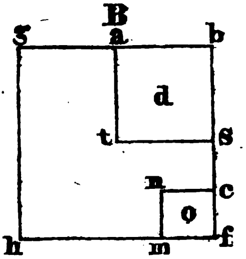
Let figure A represent the garden mentioned in the last question. The garden being in the form of a square, the length is 7 rods.

A section on one side, one rod in width, contains 7 square rods, and in the whole garden there are 7 such sections, consequently multiplying 7 by 7, the product will be the number of square rods contained in the whole garden.

Since the length of one side of a square multiplied by itself, produces the whole contents of that square, the length of one side of any square is a root, and the contents of that square the 2d power of the same root. It is from this circumstance that the 2d power of a root is called a square.

It may further be observed, that the root consists of dimensions in long measure, but the 2d power is in the dimensions of square measure.

It has already been shown, that the product of a proper fraction multiplied by a proper fraction, is less than the given multiplicand, or multiplier, consequently, the square or any power of a proper fraction, must be a less part of an unit than itself.



Let figure B represent a square surface of any kind, as a piece of board or paper, having $g\,b$, $b\,f$, $f\,h$, $h\,g$, each one foot in length, the whole surface would contain one square foot. If the side of a square be $\frac{1}{2}$ of a foot in length, we still find the area by multiplying the length of one side by itself. The square of $\frac{1}{2}$ is $\frac{1}{4}$. If $b\,s$ be half of $b\,f$, it is $\frac{1}{2}$ of a foot, and multiplying the length of $b\,s$ into itself, gives the area of the small square d, which is $\frac{1}{4}$ part of the whole square B. It may be proper to observe, that the square of $\frac{1}{2}$ of a foot in long measure is $\frac{1}{4}$ of a square foot, since d is $\frac{1}{4}$ part of B. Next let us suppose c f to be $\frac{1}{4}$ of a foot in length, and o a square, it can be only $\frac{1}{16}$ part of the whole square B, for $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$ the square of $\frac{1}{4}$. When the numerator of a fraction has more than one fractional unit in it, the square of it will be represented by a number of small squares equal to the number of fractional units in the 2d power of the given fraction, but all these small squares will still be less than a square of an unit in the same measure. It will be useful for the learner to square a number of fractions whose numerators have more than one fractional unit, and draw figures corresponding to them.

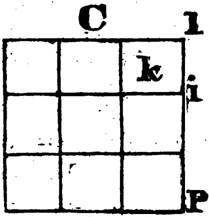
30. How much land in a square field whose side is 134 rods? Ans. $112\frac{9}{40}$ acres.

31. How much land in a yard in the form of a square, if the side is $\frac{1}{39}$ of a mile?

Ans. $\frac{1}{1521}$ of a square mile, or $\frac{640}{1521}$ of an acre, or $67\frac{493}{1521}$ s. rods.

A cubick body, or cube, is a body with 6 equal sides, or faces, the opposite sides being parallel to each other.

32. What is the number of cubical or solid feet in a cubical block, the length of whose side is 3 feet?



Let the figure C represent a cubical block 3 ft. long, 3 ft. wide, and 3 ft. thick. It was proved (Illustration page 97,) that the number of solid feet in 1 foot of the thickness on one side of a solid body, is equal to the number of square feet on that side; therefore if we square 3, the square will represent the number of solid feet in one foot of the thickness of the supposed block C; and since the thickness of a cube is equal to the length, there will be 3 sections of the block, each containing 9 solid feet; consequently multiplying 9 the square of 3 by 3, will give the number of solid feet in the whole block which is 27: Or *in other* words, the cube of the length of one side of a cubical body, expresses the whole contents of that body.

We supposed *l p* to be 3 ft. in length, which is 1 yard. Let *l i* be 1 foot in length, and it is $\frac{1}{3}$ of a yard. Now if we wish to find what the cube of $\frac{1}{3}$ of the length of the side is, we must cube $\frac{1}{3}$, which is $\frac{1}{27}$ of a solid yard, or 1 solid foot, since 27 solid feet make 1 solid yard. The cube of $\frac{1}{3}$ would be a block each side of which would be equal to *k*. Raising the length of one side of a cubical body to the third power, will always give the solid contents of that body, for the same reasons which have just been explained. This explains the reasons why the third power of a number is called the cube of that number.

33. How many cubical feet in a cubical vessel, whose side is 14 feet in length?

Ans. 2744.

34. How many solid feet in a cubical block, the side of which measures 14 inches in length?

Ans. 1 solid foot 1016 solid inches, or $1\frac{127}{216}$ cubical feet.

EVOLUTION, OR EXTRACTING ROOTS.

Involution is the method of finding a required power from a given root; *evolution* is finding a root of a given power; in one we involve or mix numbers, in the other unfold them.

If a power be divided by the root, and that quotient by the root, and so on, until the number of divisions should equal the number of times the root was multiplied into itself to create the power, the last quotient would be the root. But as the root is not supposed to be known, a method has been invented for finding it. Were the number known which produced the power, the root would be known, which would render an operation useless.

The square of any single figure cannot exceed two places of figures, since the square of 9 is only 81; the square root therefore of any three places of figures cannot be expressed by one figure. The square of any number can have no more than twice as many places of figures as the root, for the product of the left hand figure in the multiplier can never extend farther to the left of the multiplicand, than the number of places in the multiplicand. From these facts it appears, that the number of places in the root of the 2d power, when the 2d power has an even number of places, can never exceed half the number of places in the power; and when the number of places in the power is odd, the number of places in the root must be one more than half the greatest even number of places.

RULE,

For extracting the Square Root.

1. Place a point over every other figure beginning with units' place, whether the number be a mixed or whole number.

2. Find the root of the greatest square in the left hand period; subtract the square of that root from the period, and to the remainder annex the next period.

3. Double the root already found for a divisor, and ascertain how many times the divisor can be taken from the dividend, (formed by annexing the next period,) omitting the right hand figure; place the figure representing the number of times for the next figure of the root; also annex the same figure to the divisor.

Multiply the divisor by the last figure placed in the root, and subtract the product from the dividend. To the remainder bring down the next period, if any, and proceed as before.

Note.—The most convenient method of doubling the root for a divisor after the first has been found, is to double the right hand figure of the last, which may be continued for a divisor by making this increase.

1. What is the square root of 1225?

Operation.

1225(35 Ans.
9
65)325
325

Explanation.—9 is the greatest square in 12 the left hand period, we therefore subtract 9 from the period 12, and place the root, which is 3, to the right hand for the first figure of the root. We then bring down 25 the next period, and double 3 the first figure of the root for the left hand figure of the divisor. 6 can be taken from 32 five times, we therefore place 5 for the next figure of the root, and to the right hand of the divisor, then multiply the whole divisor by the last figure in the root.

2. What is the square root of 14641?

14641(121 Ans.
1
22)46
44
241)241
241

3. What is the square root of 55225?
Ans. 235.

4. Required the square root of 452929.
Ans. 673.

5. Required the square root of 1234321.
Ans. 1111.

Note 1.—When the divisor cannot be taken from all the figures except the right hand one, a cipher must occupy the next place in the root.

6. What is the square root of 1002001 ?

$$\begin{array}{r l} & \dot{1}\;0\;\dot{0}\;2\;\dot{0}\;0\;\dot{1}\;(\;1\;0\;0\;1\quad \text{Ans.} \\ & 1 \\ \hline 2\,0\,0\,1) & 0\;0\;2\;0\;0\;1 \\ & \quad\;\;\; 2\;0\;0\;1 \\ \hline \end{array}$$

7. What is the square root of 102'030'201 ?
Ans. 10101.

Note 2.—If there be a remainder, the operation may be continued by annexing two ciphers to each remainder, and the figures placed in the quotient, after the ciphers are annexed, will be decimals.

8. What is the square root of 2672 ?
Ans. 51,691+.

Illustration of the principles on which the foregoing method of extracting the square root, is founded.

When any number of square surfaces are placed in the form of a square, the number of small squares on one side, that is, the length of the whole square, is the square root of the whole number of small squares given. (See page 252.) Therefore in extracting the square root of a number, we are to find a number, which multiplied into itself, will produce the given number; or otherwise, we are to find the length of one side of a square, when the contents of that square are given.

Let us suppose we have 169 tiles,* each 1 foot square, and that we wish to know how long the side of a square platform would be, made by the same. To ascertain the length of one side of the supposed platform, we must extract the square root of 169, the whole number of tiles, for 169 would be the contents of the whole square, and the length of one side, the root.

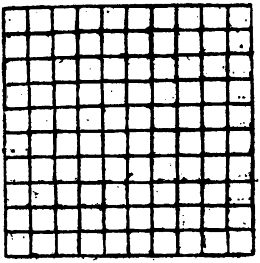
---

* A tile is a large brick having a square form.

1st part of the operation.	Operation completed.
$\dot{1}6\dot{9}(10$	$\dot{1}6\dot{9}(13$ the square root of 169.
100	1
20)69	23)69
	69

Since the whole number consists of 3 places of figures, there must be two places in the root. The left hand period is 1, or as it stands in the third place, is 100, and the greatest square in 100 is 100, and the square root of 100 is 10. We subtract the square, 100 and place 10 the root to the right hand, as may be seen in the first part of the operation. We have now taken away from the whole number of supposed tiles 100, which may be supposed to be arranged into a square form as in fig. 1. This fig. is composed of 100 small squares, and has 10 the square root of 100, on each side. If we multiply 10 by itself, the product would be 100, that part of 169 of which we have found the root. The 69 tiles which remain, must be placed on two sides of the square already formed, which meet in the same point, so that the form of the whole may still be a square.

FIG. 1.



In the full operation, we find the root of the left hand period first, and call it 1, but which is actually 10, since another figure is to be placed to the right of it. By subtracting the square of 1 from the left hand period, we actually take 100 from the whole number, since we take it from the third place. Now if we add one row to two sides of fig. 1st, the number of tiles required will be twice as many as that *part of* the root already found, and as the first figure of the root is 10 the number of tiles required to make one row on a side of figure 1st, will be 10, and to make a row on two sides the number required will be 20, twice 10. Therefore, as many times as we can take 20, double the first figure of the root, from the number of tiles remaining, so many rows each consisting of 10 tiles, can we add to each of two sides. This explains the reason for doubling the root for a divisor. In fig. 2d, let the part A, be just as large as the whole of figure 1st, then the parts *a b* and *a c* will represent the number of tiles it will require to add three rows to each of two sides of a figure as large as fig. I. which is 60, just three times 20, the divisor found by doubling the root in the first part of the operation. As 20 the first part of the divisor can be taken 3 times from 69, the remaining number of tiles, the next figure in the root must be 3. Multiplying 20 by 3, the product expresses tiles enough to complete the parts *a b*, *a c*; but this does not leave the figure in the form of a square, the corner *a* is still deficient. The side of the small square *a*, which is still wanting in figure 2d, will always be the same number as the number of rows added to two sides of A, and

Fig. II.

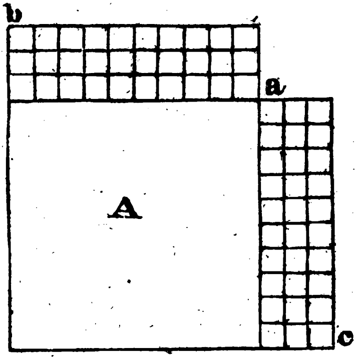
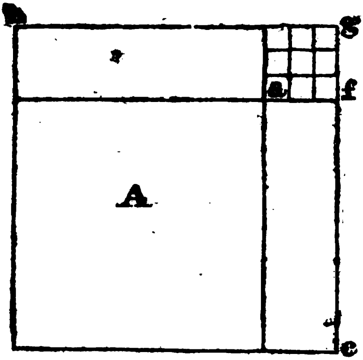


FIG. III.



if we square the number of rows added to one side, the square will express as many tiles as the small square *a* will contain.

In the full operation, before we multiply 20 by 3, the last figure in the root, we place the 3 to the right of the 2 tens, and multiplying the 3 by the 3 in the root, gives tiles enough to fill the corner *a*, and multiplying the 2 tens by the 3, gives enough to fill the parts *a b* and *a c*, as may be seen in fig. 3, where the part A is supposed to be equal to the whole of fig. 1. In fig. 3, the length of *c f* is represented by 10, the first figure of the root, and *f g* by 3, the second figure, consequently, *c g* must be 13, which shows that the number of tiles on one side of the platform, would be 13, that is, the length of one side, 13 feet.

Now if we multiply 13 feet by 13 feet, the product will be 169 feet, the number of tiles given; which shows that the root has been correctly extracted. If there were another period of figures to bring down, we should proceed in the same manner as we have in the last period, and add to two sides of fig. 3, as we now have to fig. 1.

As the length of the side of a square multiplied into itself, produces the contents of the square, or the second power of the root, the learner can easily determine whether he has worked the question correctly, by squaring the root, which will give the given number, if the operation has been rightly performed. *If there* be a remainder, the root is not exact; to obtain the power in that case, the remainder must be added to the square of that part of the root obtained.

When the root cannot be exactly expressed by numbers, it is called a *surd*.

9. If the contents of a square field be 3844 square rods, what is the length of one side ?

Ans. 62 rods.

10. If a field, in the form of a triangle, contain 8 A. 2 R. 9 r. what would be the side of a square field containing the same number of acres ?

First reduce the whole to square rods.

Ans. 37 rods.

11. What is the length of the side of a square field, which contains 10 acres ? Ans. 40 rods.

$\sqrt{\ }$ this character set before a number with an index over it, denotes that such a root of the number is to be extracted as is expressed by the index. $\sqrt[2]{\ }$ implies that the square, or 2d root, of the number after it, is to be extracted ; or $\sqrt[2]{36}$ stands for the square root of 36. $\sqrt[3]{\ }$ denotes the cube root of a number ; and $\sqrt[4]{\ }$ the biquadrate or 4th root. The index 2 is generally omitted ;—$\sqrt{16}=4$.

12. $\sqrt{81}$ is what number ? Ans. 9.

13. What number equals $\sqrt{49}$? Ans. 7.

14. What number is $\sqrt{7{,}29}$? Ans. 2,7.

15. What number equals $\sqrt{125{,}44}$? Ans. 11,2.

16. What number equals $\sqrt{376}$? Ans. 19,39+.

17. What number is equal to $\sqrt{267'970'076}$? Ans. 16'369,7+.

18. What is the square root of $\frac{1}{4}$?

Explanation.—It was shown in the *Explanation* of fig. B, page 244, that when we square $\frac{1}{2}$ of the length of the side of a square, the square, or 2d power of the root $\frac{1}{2}$, would be $\frac{1}{4}$ of the whole square, consequently, the square root of $\frac{1}{4}$ must be $\frac{1}{2}$.

square of 20, the length of the hypothenuse, will leave the square of the distance between the sill and the bottom of the ladder. Ans. 12 feet.

34. If the ridge of a building be 8 feet from the beams, and the building be 32 feet wide, of what length must the rafters be? Ans. 17,88+feet.

35. If one end of a ten feet pole be placed 6 feet from the end of a sill, how far from the end of the adjoining sill, must it reach, so that the angle formed by the two sills, shall be a right angle, or to form a square corner? Ans. 8.

36. How long a brace will be required to support a beam, if it be inserted in the post and beam just 4 feet from the point where they meet?

Ans. 5,65+.

* * *

CUBE ROOT.

It has been shown that the last product of a number multiplied into itself twice, is the third power or cube of that number; and that the number multiplied into itself twice, is the root or first power. The cube of no single figure can be more than three places of figures, for the cube of 9 the greatest single figure, is only 729. The cube of 99, the greatest number denoted by two figures, is 970'299, six places of figures. The cube of 10, the least number expressed by two places, is 1000, four places of figures. 10 is multiplied into itself twice to produce 1000; 9, twice to produce 729; and 99, twice to produce 970'299; therefore 10 is the cube root of 1000; 9 the cube root of 729, and 99 the cube root of 970'299. If any number of 9's, (which would be the greatest number that could be expressed by the same number of figures,) be cubed, there would not be more than three places of figures for each figure in the root.

From what has been said, it appears that the cube root of any three figures must always be less than two places of whole numbers; also that the cube root of any number of figures more than three and less than seven, must be two places of whole numbers besides the fraction, if any occur. From these facts, it is manifest there must be as many places of whole numbers in the cube root of any number, as there can be periods of three figures each in the given number.

RULE,

For finding, or extracting the cube root of any number.

1. Point the given number into periods of three figures each, by putting a point over units' place and every third place towards the left. If there be decimals, point towards the right in the same manner.

2. Subtract the greatest cube contained in the left hand period from that period, and set the root of that cube to the right of the given number: Annex the next period to the remainder for a dividend.

3. Take three times the square of the root already found for an imperfect divisor.* Ascertain how many times the imperfect divisor can be taken from the dividend, and place the number in the root.

4. Multiply three times the whole root, omitting the right hand figure, by the whole root, and add the square of the right hand figure, the sum will form a perfect divisor.

Multiply the perfect divisor by the last figure written in the root, and subtract the product from the dividend.

---

* It is called imperfect, because it is not so large as the real divisor, which cannot be found until the next figure of the root is found.

5. To the remainder annex the next period, and use the last perfect divisor as an imperfect divisor for the new dividend, and proceed as before directed.

Note 1.—When the right hand figure is omitted, to multiply by 3, put a cipher in its place. If the right hand place be a cipher, it is not to be omitted.

1. What is the cube root of 512?

Explanation.—When only three places of figures are given, the root is found by trial.

The same method must be pursued with the first period of a larger number. Ans. 8.

2. What is the cube root of 343? Ans. 7.

3. What is the cube root of 125? Ans. 5.

4. What number multiplied into itself twice, will produce 216? Ans. 6.

5. What is the cube root of 12'167?

1 2 1 6 7 (2 3 cube root of 12'167.
8

Imp. divi. 1 2) 4 1 6 7
Perf. divi. 1 3 8 9) 4 1 6 7

Manner of finding the perfect divisor.

2 0 whole root except 3.
3 [except the 3.
6 0 three times the whole root.
2 3 whole root.
1 8 0
1 2 0
Product of 3 times the whole root except 3 into the whole root 1 3 8 0
9 square of the right hand fig.
1 3 8 9 perfect divisor.

Note 2.—The first imperfect divisor is hundreds when considered in connexion with the dividend, the two right hand figures of the dividend should therefore be omitted in ascertaining how many times it can be taken from the dividend

It will also be proper to observe here, that any imperfect divisor is so much smaller than the perfect one, that the next figure to be obtained in the root, will not be so large as might at first be supposed by comparing it with the dividend.

6. What is the cube root of 1'860'867?

		$\dot{1}$ 8 6 $\dot{0}$ 8 6 $\dot{7}$(123 Ans.
		1
1st imperfect divisor	3	8 6 0
1st perfect divisor	3 6 4	7 2 8
2d imperfect divisor	3 6 4	1 3 2 8 6 7
2d perfect divisor	4 4 2 8 9	1 3 2 8 6 7

7. What is the cube root of 32768? Ans. 32.

8. Extract the cube root of 79507. Root, 43.

9. Required the cube root of 175616. Ans. 56.

10. What number equals the cube root of 300763? Ans. 67.

11. What number multiplied twice into itself, will make 343000? Ans. 70.

12. What number raised to the third power equals 512000? Ans. 80.

13. Of what number is 729000 the cube? Ans. 90.

Note 3.—When a remainder occurs, the operation may be continued by annexing periods of three ciphers each. The figures obtained in the root after annexing the ciphers, will be decimals.

When the cube root of a vulgar fraction is required, the cube root of the numerator will be the numerator of the root, and the cube root of the denominator, the denominator, if the operations can be performed without a remainder to either; if not, the vulgar *fraction must be changed to a decimal.*

14. Of what number is 277690 the third power?

Ans. 65,24+.

15. What is the cube root of $\frac{1}{343}$? Ans. $\frac{1}{7}$.

16. What fraction equals the cube root of $\frac{64}{729}$?

Ans. $\frac{4}{9}$.

17. Extract the cube root of $\frac{2197}{4913}$. Ans. $\frac{13}{17}$.

18. What is the cube root of $\frac{3}{13}$?

$\frac{3}{13}$=,230769+ ; and $3\surd$,230769=,61+, Ans.

19. What is the cube root of $\frac{1}{5}$?

Ans. ,584+.

Illustration of the principles on which the foregoing method of extracting the cube root, is founded.

The 3d power of a number is a number expressing the cubical or solid inches, feet, &c. contained in any body in the form of a cube; the cube root of that number is the length of one side. (See page 245.) When the solid contents of a cubical body are given, or any number of cubical inches, feet, &c. to be arranged in the form of a cube, to find the length of one side we extract the cube root.

To illustrate that operation by the rule, will give a result corresponding with what has been said of the cube root, let us suppose we have 2744 cubical blocks of wood, each containing 1 solid inch. If these blocks be placed together, so that the number of blocks in the length, breadth, and thickness shall be equal, the pile would be in the form of a cube, and the number of blocks it is high, wide, or long, that is, the number of inches, will be the cube root. In 2744 there are two periods, consequently there must be two places of figures in the root.

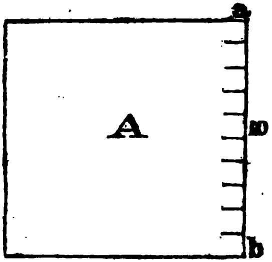
Operation.

```
     .     .
     2 7 4 4 ( 1 4
     1
     ---------
 3 ( 1 7 4 4
 ----1 7 4 4
436 ---------
```

The greatest cube in 2, the left hand period, is 1, and the root 1; or the greatest cube in 2000 is 1000, and the cube root of 1000 is 10, denoted by the first figure in the root. By subtracting 1000 from the whole number, we suppose blocks enough taken to make a cubical pile, whose length, breadth, or thickness is 10 inches, since 10 multiplied into itself twice, produces 1000, the number of blocks already taken from the whole number, and which form the supposed cube, whose side is 10 inches.

In fig. 1, let the square A represent one side, or face, of the cubical pile made by the 1000 blocks; then the side *a b*, which is equal to each of the other sides, will be 10 inches in length. 1744 blocks still remain to be added to the pile A. To keep the pile in the form of a cube, the additions of the remaining blocks must be made to three sides. The next object is to find how many blocks will be required to cover those three sides, one block, or inch thick. This we shall do by ascertaining how many square inches there are on the three sides.

Fig. 1.



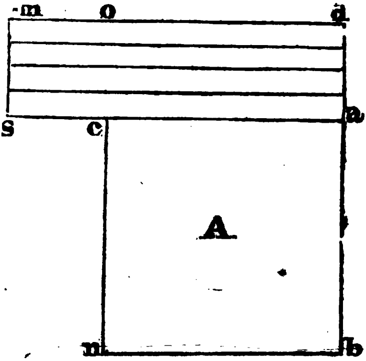
1st part of operation.

```
 .     .
 2 7 4 4 ( 1 0
 1 0 0 0
 -------
300)1 7 4 4
```

The square of *a b*, or 10 inches, is 100 square inches, the number of square inches on the whole face A; and 3 times 100 are 300, the number of square inches on the three faces to which the three additions are to be made.

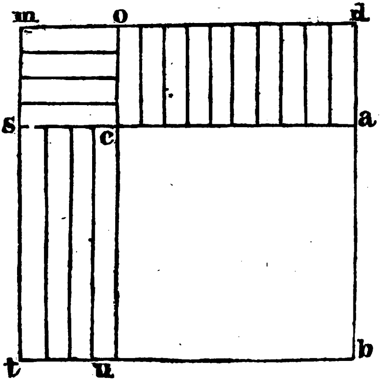
300 is used as an imperfect divisor, since as many times as 300 can be taken out of the remaining number of blocks, so many blocks or inches in thickness, can be added to 3 faces, each equal to A. It appears

Fig. ii.



by the first part of the operation, that 300 can be taken 5 times from 1744, but by adding to one side, the other sides are increased, so that it will require more than 300 blocks to cover 3 faces each larger than A. In fig. 2, let the part A represent the face of the cube formed with the 1000 blocks, the same as in fig. 1. Now if we add several thicknesses to the side *a c*, the sides *d b* and *o n b d* are each larger than the part A. A perfect divisor must be a number expressing the number of square inches on the three sides to which the additions are to be made, after the sides are increased, 300 is therefore too small for a perfect divisor. Let us suppose the real divisor can be taken 4 times from the remaining number of blocks, then the whole length of one side of the block, after the additions are made, must be 14 inches. In fig. 2, the part *a s* represents one layer of blocks added to the side *c a* and extending to *s*. If the additions can be made 4 inches thick, *s c* must be 4 inches long, since *c a* is 10; and since it is 10 inches from *a* to the bottom of the pile, the layer *s a* must be 14 inches long, and 10 inches wide.

FIG. III.

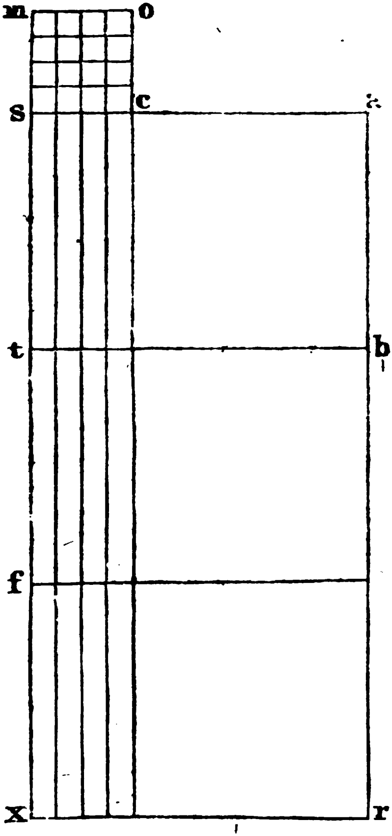


By examining fig. 3, it will be seen, that we shall have 3 sides, each 14 inches long, and 10 inches wide, besides a small face 4 inches square, upon which additions are to be made to complete the cube.

Let *c u* represent a layer extending to the bottom of the pile, and rising 4 inches higher than the part *a b* in fig. 2; if *c u* be 10 inches, that layer will be 14 inches high and 10 inches wide. Let *s c* and *a d* be each 4 inches; also, let it be 10 inches from *a* to the bottom of the pile, then *s a* will be 14 inches long and 10 inches high, and *o u b d* 14 inches long and 10 inches wide. Now if the additions be made 4 inches thick, the corner *m o c s* will be 4 inches square, which is not included in either of the other parts to which the additions are to be made. The number of square inches contained in the three faces *c u*, *s a*, *o u b d*, which have all been shown to be equal, added to the number contained in *m o c s*, will constitute the perfect divisor, since by adding to them, the form of the pile will be a cube. The width of each of the large faces is 10 inches, and the length 14.

In fig. 4, let *m o c s* equal *m o c s* in fig. 3, and *a b* be 10 inches, *a s* 14, and all the angles right ones; also, let *b f* and *f r* each equal *a t*, the whole figure will represent the whole surface upon which the additions are to be made. *a b* is 10 inches, the whole root, except the right hand figure, and *a r* 3 times that number, but *a s* or *x r* represents 14 inches, the whole root.

Fig. IV.



Now if we multiply the length *a r*, 30 inches, that is, 3 times the whole root, except the right hand figure, by *s a*, 14 inches, the whole root, the product will be the number of square inches contained in *a s x r*, and the square of *m o*, which is 4 inches, the last quotient figure, the number contained in the small square *m o c s*; therefore, three times the whole root, except the right hand figure, multiplied by the whole root, and added to the square of the last quotient figure, will always form a perfect divisor, as this will always be the case, let the length of *a r*, *a s*, and *m o*, be any other corresponding numbers. By this method, in the present example, we find the perfect divisor to be 436, the number of square inches contained in *m o c s* and *s a r x*; and if we multiply the number of square inches contained in these surfaces by the thickness of the additions, the product will express the number of blocks required to complete the additions.

Operation completed.

```
      2 7 4 4 ( 1 4
      1
      -------
436 ) 1 7 4 4
      1 7 4 4
      -------
```

436 can be taken 4 times from 1744, the remaining number of blocks, therefore the last figure of the root will be 4, and the length of one side of the cube formed by the whole number of blocks, 14 inches, the cube root of 2744. When the number of periods are more than two, the other additions are made in the same manner.

Note 4.—It will be difficult for young students to understand this demonstration without the aid of blocks. Every instructer should be furnished with them.

20. If 59'876'471 cubical blocks, each one foot on a side, be placed in the form of a cube, what will be the length of one side?

Ans. 391 feet.

21. How long is the side of a cubical box, which will contain 110'592 cubical feet of grain, if 3 inches be allowed for the thickness of the sides of the box? Ans. 48 ft. 3 in.

22. If the earth excavated in digging a trench one mile in length, 40 feet wide, and 6 feet deep, be thrown into a cubical form, how high will the cube be, allowing the earth to occupy $\frac{7}{8}$ of the space which it did before it was removed? Ans. 103,5+feet.

23. If a cubical box contain 71 cubical feet, what will be the length of a cubical box which shall contain 3 times as much? Ans. 5,97+feet.

24. What is the cube root of 9'999'999'999'000'-000? Ans. 215443,46+.

Remainder to question 24th is 1'252'672'906,-066264.

25. What is the height of a cubical box, which will contain nine times as much as one whose height is ten feet? Ans. 20,8+feet.

26. If the length of one side of a cubical block be 1 inch, what will be the length of one which will contain half as much? Ans. ,79+in.

27. What is the cube root of one thousand million? Ans. One thousand.

28. What is the cube root of 1? Ans. 1.

29. If the diameter of a globe be 15 inches, what will be the diameter of a globe twice as large?

Explanation.—Multiply the cube of the given diameter by 2, and extract the cube root of the product. Ans. 18,89+inches.

30. If the diameter of a globe be 2, what is the diameter of another half as large? Ans. 1,58+.

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SECTION X.

PROGRESSION.

DEFINITIONS.

A Series of Numbers is several numbers following each other in succession.

Progression, is a series of numbers regularly increasing or decreasing.

Arithmetical Progression is a series of numbers increasing or decreasing regularly by the constant adding or subtracting of some particular number. 2, 4, 6, 8, 10, 12, is a series of numbers in arithmetical progression, made by the constant addition of the number 2; or, 15, 12, 9, 6, 3, is a series decreasing by the constant subtraction of 3.

Common Difference, the number added or subtracted to make out the series.

Extremes, the first and last terms of a series.

Means, the terms between the extremes.

ARITHMETICAL PROGRESSION.

PROBLEM 1.

1. If a man work 10 years, and clear 100 dollars the first year, 111 the second, and so on, clearing 11 dollars each year more than he did the preceding one,—how many dollars did he clear the 10th year.

$$11\times9=99+100=199.$$

Ans. $199.

Illustration.—By adding 11 together 9 times, the sum is the number of dollars he would gain more than the 100, the tenth year, and by adding the 100, the sum is his whole gain the tenth year; but if we multiply 11 by 9, the product will be the same as the sum found by adding. When a series of numbers in arithmetical progression, is made out at full length, the common difference is added to, or subtracted from all the terms except the last one; consequently, the last term in an increasing series, or the first term in a decreasing, contains the common difference as many times as the series has terms wanting one. But as the product of a number, multiplied by any number of units, is equal to the multiplicand added together as many times as the multiplier contains units, we obtain the same number whether we add the common difference as many times as the series has terms, wanting one, or multiply it by the same *number.*

RULE,

For finding the last term.

Multiply the common difference by one less than the number of terms, and add the first term to the product, if the series be increasing ; but if the series be decreasing, subtract the product from the first term.

Note.—All questions may be considered to be in increasing progression, unless mentioned otherwise in the question.

2. What is the last term of a series, which has 1001 terms, a common difference of 7, and the first term 19 ? Ans. 7019.

3. Given the first term of a series 111'111, the number of terms 1'111, and the common difference 11 ; what is the last term, if the series decrease ? Ans. 98901.

4. If a vessel sail 100 days successively, and sail 100 miles the first day, how far will she sail the last day, allowing she decreases each day's sail $\frac{1}{3}$ of a mile ? Ans. 67 miles.

The following examples are given to find the last terms.

	First terms.	Com. dif.	No. terms.	Ans.
5.	20202,	5,	202,	21207.
6.	60000,	10,	100,	60990.
7.	675,	$\frac{1}{4}$,	224,	730$\frac{3}{4}$.
8.	1909,	$\frac{1}{5}$,	20,	1912$\frac{4}{5}$.
9.	821,	$\frac{2}{7}$,	28,	828$\frac{5}{7}$.

PROBLEM II.

To find the sum of the terms.

10. A man paid a debt at 41 several payments. The first payment was $26,01, and each succeeding payment increased in arithmetical progression with a common difference of $1,85 ; what was the sum of all the payments, that is, what was the debt ?

Here 185 cents×40=7400 cents; and 7400 cents+2601 cents=10001 cents the last payment. Then 10001 cents+2601=12602 cents the sum of the first and last payments, and 12602×41, the number of payments=516682 cents double the sum of all the payments. Ans. $2583,41.

Illustration.—The sum of any two terms of an arithmetical series equally distant from the centre, is equal to double the sum, which all the terms will average; or if the number of terms be odd, equal to double the middle term, consequently multiplying the sum of the extremes by the number of terms, gives double the sum of the series.

RULE.

Multiply the sum of the extremes by the number of terms, and divide the product by 2; or multiply half the sum of the extremes by the number of terms.

11. If the first term of an arithmetical series be 2, the last term 122, and the number of terms 29, what is the sum of all the terms? Ans. 1798.

12. What is the sum of 1000 terms in arithmetical progression,—the last term being 500, and the common difference $\frac{1}{6}$, if the series decrease? Ans. 583250.

13. Required the sum of 990 terms of the numbers 2, 5, 8, 11, &c.? Ans. 1470645.

Given the following examples to find the sum of the series in each.

	First terms.	Last terms.	No. Terms.
14.	- 1'011,	28,	66,
			Ans. 34'287.
15.	1'000'000,	999,	555,
			Ans. 277'777'222$\frac{1}{2}$.
16.	96,	$\frac{3}{16}$	23,
			Ans. 1'105$\frac{37}{32}$.

Required the sum of the series in the following examples.

	First terms.	Com. diff.	No. terms.	Ans.
17.	50,	2,	26,	1950.
18.	201,	$\frac{1}{6}$,	29,	$5896\frac{2}{3}$.
19.	$\frac{9}{11}$,	$\frac{1}{91}$,	101,	$138\frac{131}{1001}$.

Find the sum of all the terms in the following examples, in decreasing arithmetical progression.

	First terms.	Com. diff.	No. terms.	Ans.
20.	1000,	3,	38,	35891.
21.	120,	6,	11,	990.
22.	1,	$\frac{1}{49}$,	40,	$24\frac{4}{49}$.
23.	10,	$1\frac{1}{2}$,	5,	35.

PROBLEM III.

To find the common difference.

RULE.

Divide the difference of the extremes by one less than the number of terms.

Illustration.—The greater term is made up of the less, and the common difference taken as many times as the series contains terms wanting one; therefore by subtracting the less from the greater extreme, and dividing the difference by one less than the number of terms, the quotient will be the common difference of the terms.

24. The extremes of an arithmetical series are 3 and 97, and the number of terms 48; what is the common difference? Ans. 2.

25. If a man perform a journey in 8 days, increasing each day's travel by a certain number of miles, and travel the first day 21 miles, and the last 70;—how many miles does he increase daily? Ans. 7.

PROBLEM IV.

To find the number of terms.

RULE.

Divide the difference of the extremes by the common difference, and add one to the quotient.

Illustration.—The greatest term is made up of the ommon difference added together as many times as the series has terms wanting one, together with the less term; therefore by subtracting the less term from the greater, and dividing the remainder by the common difference, the quotient expresses the number of times the common difference was added together to make the dividend, which is the difference between the extremes.

25. The extremes of an arithmetical series are 8 and 88, the common difference 10; what is the number of terms? Ans. 9.

26. If a boy buy a number of oranges, and give 17 cents for the first, $15\frac{1}{2}$ cents for the second, &c. decreasing the price of each, $1\frac{1}{2}$ cents until the last cost but 2 cents;—how many oranges did he buy? Ans. 11.

27. A spendthrift spent his estate, amounting to $2873,10, in 15 years,—spending each year $20,22 more than in the preceding; how much did he expend the 9th year? Ans. $211,76.

Questions to be performed by the principles contained in the preceding problems.

28. If two men start from the same place at the same time, one travelling 21 miles each day, the other 3 miles the first day, 5 the second, increasing his speed each day by the excess of two miles; how many days before the latter will come up with the former,—and how many miles will each travel?

Ans. { 19 days.
399 miles.

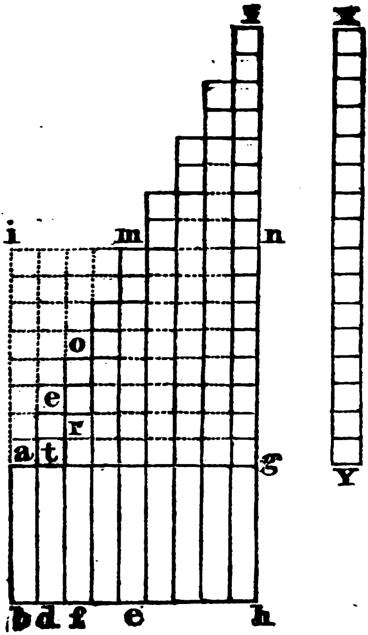
29. Estimating the distance between London and Boston at 3000 miles, if a ship perform a voyage from one city to the other in 26 days,—sailing the first day 80 miles, and increase each day's sail in arithmetical progression; how far will she sail the last day; what will be the daily increase; and how much farther will she sail the 25th day of the voyage than the second?

Ans. { Last day's sail $150\frac{10}{13}$ miles.
Daily increase $2\frac{54}{65}$ miles.
25th day's sail more than the 2d,
$65\frac{7}{65}$ miles.

30. Two divisions of labourers commence work upon a canal at the same time, 500 rods apart; one division completes one half of the intervening distance in 7 days; the other the remaining half in 9 days. Now if each division increase each day's work 4 rods more than the last, how many rods does each division complete in each day?

Ans. { The first, $23\frac{5}{7}$, $27\frac{5}{7}$, $31\frac{5}{7}$, $35\frac{5}{7}$, $39\frac{5}{7}$, $43\frac{5}{7}$, and $47\frac{5}{7}$.
The second, $11\frac{7}{9}$, $15\frac{7}{9}$, $19\frac{7}{9}$ $23\frac{7}{9}$, $27\frac{7}{9}$, $31\frac{7}{9}$, $35\frac{9}{7}$, $39\frac{7}{9}$, and $43\frac{7}{9}$.

The methods which have been explained for finding the last term, sum of the terms, common difference and number of terms, may be more clearly illustrated by the assistance of the adjoining plate.



Suppose we have 9 piles of equal blocks, which increase by 2, and that the first pile consists of 5. Let *a b* represent the first pile, *c d* the second, &c. It has been observed, that we shall find the last term by multiplying the common difference by one less than the number of terms, and adding the first term to the product. By examining the plate, it will be seen that 2 blocks are added to *t d*, which is equal to *a b* the first pile, to make *c d* the second, and *r o* added to *r f*, which is equal to the second pile, to make *o f* the third. In the same manner all the others are formed.

As we do not add to make the first pile, we can add 2 only 8 times, which is once less than the number of terms to make the last, or the 9th pile. Now if we suppose *c t*, *o r*, &c. to be removed on to the column *g l*, as all the small squares are supposed equal, they would just fill that column, as is now represented, which represents 16 blocks, the number found by multiplying 2 the common difference, by 8 one less than the number of terms; and if *a g* and *b h* be parallel, *g h* is equal to the first term *a b*, and *l h* will represent the 9th pile or last term of the series. From this it appears that the last term of an increasing series, will always be made up of the common difference taken as many times as there are terms wanting one and the first term, for we may suppose *a b* to be any other number than 5, and *g l* will always be made up of the common difference, whether it be 2 or any other number.

2. If all the blocks represented by the small squares above the line *m n*, be removed and placed upon the dotted squares, there would be just enough to fill them, and *i n h b* would represent all the blocks contained in the 9 piles, that is, each pile would average the same number as is contained in the middle one *m e*; and if we multiply *m e* or *n h* by *h b*, the product would express the number of blocks contained in all the piles; but if we add *n h* and *i b* together, the number would be twice *n h*, and multiplying twice *n h* by *h b* the number of piles, would give twice the whole number of blocks, or twice the sum of the series. It should be observed, that the number of blocks contained in *n h* and *i b* represent twice the number contained in the middle term *m e*.

3. Let the number of squares in *x y* equal the number in *l g*, then *x y* represents the number of

blocks added after the first term is subtracted. If the number contained in $x\ y$ be divided by the number of times the additions were made, the quotient would be the number added, that is, the common difference; and if the number be divided by 2, the quotient would be 8, the number of times the common difference was added, which is once less than the number of terms.

◆

ARITHMETICAL PROPORTION.

When two numbers are compared together to ascertain how much larger one is than the other, or how many times one contains the other, the first is called the *antecedent*, the second the *consequent*; both taken together, a *couplet*.

If the difference between the antecedent and consequent of two or more couplets, be the same number, this equality of difference is termed *arithmetical proportion*, and each number, considered in connection with the others, is called an *arithmetical proportional*.

The proportion of numbers is expressed by placing a colon between the antecedent and consequent of each couplet, and a double colon between the couplets.

Thus, 2 : 4 : : 6 : 8, are four numbers in arithmetical proportion; and are read, as 2 is to 4, so is 6 to 8.

Proportion is distinguished into *continued* and *discontinued*.

When the difference between the consequent of one couplet, and the antecedent of the next, is the same number as the common difference of the couplets, the proportion is *continued*; but when the difference between the consequent of one couplet and the antecedent of the next, is not the same number as the common difference of the couplets, the proportion is *discontinued*.

Thus, 2 : 4 : : 6 : 8, is *continued* proportion; but, 2 : 4 : : 10 : 12, is *discontinued* proportion.

PROMLEM I.

To find a mean proportional between two given extremes.

RULE.

Take half the sum of the two extremes, the quotient will be an arithmetical mean proportional between the two given extremes.

1. What is the mean proportional between 7 and 17? Ans. 12.

2. A certain debt can be discharged at 3 payments in arithmetical proportion. If the first payment be $11, and the third $31,17, what is the second? Ans. $21,085.

PROBLEM II.

To find any number of arithmetical means between two given terms.

RULE.

Subtract the less extreme from the greater, and divide the difference by one more than the number of means required;—this quotient added to the first term, if the proportion is increasing, or subtracted, if it be decreasing, will give the first mean; added to, or subtracted from the first mean, will give the second.

Continue the same process till all the means are found.

3. Required to find four arithmetical means between 1 and 96. Ans. 20, 39, 58, 77.

4. If a man work 6 days for wages in arithmetical proportion, and receive 29 cents for his first day's

work and 79 for the last, what does he receive for the intervening days?

Ans. 39, 49, 59, 69 cents.

* * *

GEOMETRICAL PROGRESSION.

When a series of numbers is produced by continually multiplying or dividing by the same number, it is called *geometrical progression*. The number by which a series in geometrical progression is increased or decreased, is called the *ratio*.

Thus, 1, 2, 4, 8, 16, 32, is an increasing series in geometrical progression, having 2 for its ratio; and 81, 27, 9, 3, 1, is a decreasing series, having 3 for its ratio.

All the other terms have the same names as the corresponding terms in arithmetical progression.

PROBLEM I.

To find the last term.

RULE.

Raise the ratio to a power denoted by one less than the number of terms, and multiply that power by the first term, if the series be increasing, but if decreasing, divide by the same power.

Illustration.—When the first term of an increasing series is one, the second term will always be the same number as the ratio; consequently, multiplying by the ratio to find the last term, is only raising the ratio to a power denoted by one less than the number of terms. But when the first term is any number more than one, the second term will contain the ratio as many times as the first term contains an unit; so that the last term will contain such a power of the ratio, as is denoted by one less than the number of terms, as many times as the first term contains an unit.

Thus, in the series 1, 2, 4, 8; 8, the last term, is simply the third power of the ratio. But in the series, 2, 4, 8, 16, the last term contains twice the third power of the ratio.

When the progression is decreasing, it may be considered as increasing from the last term; consequently dividing the first term by that power of the ratio denoted by one less than the number of terms, will give the last term.

1. Let the first term of a geometrical series be 5, the ratio 3, number of terms 10; what is the last term? Ans. 98415.

2. Given the first term of a decreasing series, 1000, number of terms 5, the ratio 4, to find the last term. Last term, $3\frac{29}{32}$.

3. The first term of a series is $\frac{1}{16}$, the ratio $\frac{1}{16}$, the number of terms 4; what is the last term? Ans. $\frac{1}{65536}$.

4. A man agrees to sell 10 bushels of wheat at 1 mill for the first bushel, and 5 times as much for each succeeding bushel as what he receives for the preceding one; what will he receive for the last bushel? Ans. $1953,125.

5. What is the last term of 9 terms of the series $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, &c.? Ans. $\frac{1}{19683}$.

6. If a balloon descend from a certain height in 6 minutes, and descend $\frac{1}{125}$ of a mile the first minute, three times that distance the second, 9 times as far the third minute as the first, and increase its velocity during the remainder of its descent by the same ratio; how many feet will it descend the last minute? Ans. $10264\frac{8}{25}$.

PROBLEM II.

To find the sum of all the terms.

Illustration.—Let it be required to find the sum of the four terms in the series 1, 2, 4, 8. If we add all the terms together, we shall obtain 15 for the sum. *The second* term contains the ratio once, the third twice, the fourth 4 times, and the three last, 7 times. Now if we multiply the last term by the ratio, the product 16 will contain the ratio 8 times, once more than what is contained in the three last terms,—but as the first term is to be included in the sum, we shall obtain the sum by multiplying the last term by the ratio, and subtracting the first term. Again, let it be required to find the sum in the series 2, 6, 18.

Operation.

2, 6, 18
3
54
2
2)52
Sum of terms, 26

The ratio in this example is 3, and 54 contains the ratio 18 times; or in other words, it contains the ratio 16 times, and the first term 3 times. By taking the first term once from 54, the remainder contains the ratio 16 times, and the first term twice. As the two last terms contain the ratio but 8 times, were we to take twice the first term from 52, and divide the remainder, 48, by 2, one less than the ratio, the quotient would express the sum of the two last terms; but if we divide 52 by 2, the quotient will be two more than when we divided 48 by 2, that is, the sum of all the terms.

The last term of any increasing series multiplied by the ratio, contains the first term as many times as the last term contains an unit, and the sum of all the other terms once less, therefore multiplying the last term by the ratio, and dividing the difference between that product and the first term, will give the sum of the series. If the series be decreasing the principle is reversed.

RULE.

If the series be increasing, multiply the last term by the ratio, and divide the difference between that product and the first term by the difference between 1 and the ratio. But if the series be decreasing, multiply the first term by the ratio, and divide the difference between that product and the last term by the difference between 1 and the ratio.

Note.—When the ratio is less than one, the sum is found the same as though the series were increasing, although in reality the series decreases.

7. Find the sum of the terms in the following series: 3, 9, 27, 81. Ans. 120.

8. The number of terms of a geometrical series is 8, first term 3, and the ratio 4; what is the sum of the series? Ans. 65535.

9. A certain gentleman gave his whole estate to six sons,—giving to the youngest $1000; to the second once and a half as much, and to the others, sums increasing by the same ratio; how much was the estate? Ans. $20781,25.

Required the sums of the terms in the following examples.

	First terms.	Ratios.	No. terms.	Ans.
10.	1,	2,	9,	511.
11.	20,	3,	8,	65600.
12.	7,	$\frac{2}{3}$,	7,	$19\frac{562}{729}$.
13.	$\frac{2}{7}$,	$\frac{1}{7}$,	9,	$\frac{3954653388}{11863960458}$.

GEOMETRICAL PROPORTION.

When two or more couplets of numbers have equal quotients, or ratios, this equality is called *proportion*; or, when the antecedents of two or more couplets, are the same parts of their respective consequents, or are the same multiples of their consequents, the numbers are said to be in *proportion*; the terms themselves are called *proportionals*. Thus, in the proportion 2 : 4 : : 8 : 16, 2, the antecedent of *the first* couplet, is the same part of 4, its consequent, *that 8,* the antecedent of the second couplet, is of 16, its consequent; and the proportion, 81 : 27 : : 9 : 3, 81 is the same multiple of 27, that 9 is of 3.

When the ratio between the consequent of the first couplet and the antecedent of the second, is the same number as the ratio between the antecedent and consequent of each couplet, the proportion is said to be *continued;* but when this is not the case, the proportion is called *discontinued.*

THEOREM.

When four numbers are in geometrical proportion, the product of the extremes is equal to the product of the two means. But when the number of terms is odd, the product of the extremes is equal to the square of the middle term.

Illustration.—When four numbers are in geometrical proportion, if one extreme be 1, the proportional next to it will always be the same as the ratio; consequently, multiplying the other mean by that which is equal to the ratio, gives the greater extreme; and this product of the means is evidently equal to the product of the extremes; for multiplying the greater extreme by the less, when that is one, does not increase it.

In the proportion 3 : 9 : : 27 : 81, three times 81 is 243, the 5th power of 3 the ratio, and 27, the third power of the ratio, multiplied by 9, the second power, gives 243, the 5th power. Also, in the proportion 2 : 6 : : 18 : 54, two times 54, the product of the extremes, is 108, which is the same as 6 times 18, the product of the means, since 54 contains 18 three times. The same is equally true when the proportion is discontinued. 2 : 6 : : 7 : 21. In this proportion, 42, the product of the extremes, is made up of six 7's, which is equal to the product of 6 multiplied by 7, the two means. This will always be the case, for the fourth term always contains the third as many times as the second contains the first, or is the same multiple of it. These examples show, that the product of the extremes will always equal the product of the means.

When three terms are in proportion, and one of the extremes is 1, the mean is the same as the ratio, and multiplying the mean by itself, gives the other extreme, which is also the square of the mean. In the proportion 1 : 4 : : 4 : 16, the product of the extremes and the product of the means, each equal the square of 4, the ratio.

PROBLEM I.

To find any number of geometrical means between two given extremes.

RULE.

Divide the greater extreme by the less, and extract such a root of the quotient as is denoted by one more than the number of means required. Then, if the series be increasing, multiply the first term by the root just found, continually, but if the series be decreasing, divide the first term continually by the same root, the products or quotients will be the means required.

Illustration.—According to Illustration, page 276, the greater extreme of any number of proportionals, is made up of as many times of that power of the ratio denoted by one less than the number of terms, as the less extreme contains units, therefore dividing the greater extreme by the less, gives that power of the ratio for a quotient, and extracting such a root of this quotient as is denoted by one less than the number of terms, or, (which is the same thing,) one more than the number of means, gives the ratio.

1. Let it be required to find two geometrical means between 3 and 81.

Here $81 \div 3 = 27$, and $\sqrt[3]{27} = 3$, the ratio; then the first term $3 \times 3 = 9$, the first mean, and $9 \times 3 = 27$, the second mean.

2. Find 3 geometrical means between 5 and 80.

Means. 10, 20, 40.

3. If a merchant sell 7 yards of cloth, and receiv

prices for the same in geometrical proportion, what does he receive for the 2d, 3d, 4th, 5th, and 6th yards, if he receive 1 cent for the first yard, and $7,29, for the seventh?

Ans. 3, 9, 27, 81, 243 cents.

PROBLEM II.

Having one extreme and one or two means given to find the other extreme.

Illustration.—As the product of two means is equal to the product of the two extremes, it may be considered as the product of the two extremes; and if the product of two numbers be divided by either of those numbers, the quotient will be the other.

RULE.

When two means are given, divide their product by the given extreme. If but one mean is given, divide the square of the mean by the given extreme.

Remark. The nature of the question will generally determine which term is the extreme. As both antecedents, that is, the first and third terms are either multiplied or divided by the ratio to produce their consequents, it will at once be seen whether the third term is greater or less than that which is demanded. If greater, the greater of the two other terms, will of course, be the first term or antecedent; but if less, the first term will be less than the second.

4. What is the 4th term of four proportionals, if the product of the two means be 676 and the first term 4? Ans. 169.

5. What number must be divided by 4, that the quotient shall be the same number as the quotient of 39 divided by 13? Ans. 12.

6. 35 bears the same proportion to 37 as 9 to what number? Ans. $9\frac{18}{35}$.

7. Suppose two companies of men spend money in proportion to the number of individuals in each company, and one company to consist of 11, and the

product of the larger company into what the less company expend, equal $14595; what is the expense of the larger company?

Ans. $1326,918+.

8. What number must be divided by 31, that the quotient may be the same as the quotient of 15 divided by 3; and what number must 31 be multiplied by to produce that number?

Explanation.—The first number demanded must contain 31 as many times as 15 contains 3. If we divide 15 by 3 the quotient will express the number of 3's contained in 15, which is 5; and if we multiply 31 by 5, the product will contain 31, 5 times, as many times as 15 contains 3. 5 is the ratio, because it is the number by which 3 was multiplied to make the second term or first consequent, and 31, the second antecedent, must be multiplied by the ratio in order that the fourth term may be in the same proportion to 31 that 15 is to 3. The proportion demanded in the question, is, that 3 can be taken from 15 as many times as 31 can be taken from 155; or 3 : 15 :: 31 : 155. If we invert the terms, the numbers will still be in proportion, and have the same ratio; but the ratio becomes a divisor instead of a multiplier. Thus 15 : 3 :: 155 : 31; or 15 is the same multiple of 3 that 155 is of 31, for the two consequents are formed by dividing the two antecedents by 5, the ratio, which was the former multiplier. This is called *inverting the terms of the proportion.* It will frequently be more convenient to find the ratio by dividing the larger of the two first terms by the smaller, and multiplying or dividing the third term by it, which will give the fourth. This method of operating will always show the scholar more clearly the nature of proportion.

9. What number is the same multiple of 7 that 61 is of 9?

The proportion will be, as 9 : 61 :: 7 to the answer. Ans. 47$\frac{4}{9}$.

10. What number is the same part of 27 that 13 is of 19? Ans. 18$\frac{9}{19}$.

11. The first of three terms is 9, the square of the mean 256; what is the third proportional?

Ans. $28\frac{4}{9}$.

12. Let the product of two means be $\frac{4}{361}$, the first term $\frac{1}{5}$; required the fourth term? Ans. $\frac{20}{361}$.

13. Find a third proportional to 8 and 13.

Ans. $21\frac{1}{8}$.

14. Find a third proportional to $\frac{2}{733}$ and $\frac{13}{15}$.

Ans. $38\frac{351}{1663}$.

15. If the square of a mean proportional be $\frac{31}{33}$, and the first term be $\frac{1}{18}$; what is the last term?

Ans. $16\frac{30}{33}$.

16. What number is that which bears the same proportion to 79 that 17 does to 15?

Ans. $89\frac{8}{15}$.

17. If 15 yards of cloth cost 16 dollars, what will 90 yards cost at the same rate?

Explanation.—If we divide 16 by 15, the quotient will be the price of one yard, and 90 multiplied by the price of one yard, will give the price of 90 yards. The price of one yard is the ratio, therefore 15 must be to its price 16 dollars, as 90 is to 96 dollars its price. Or if we multiply 90 yards by 16 dollars, the price of 15 yards, the product must contain the price of 9 yards 15 times; consequently dividing that product by 15 will give the true answer.

18. If a man sell 2 pounds of butter for 31 cents, what will 1000 pounds come to? Ans. $155.

19. If a horse eat 3 tons of hay in 7 months, how much would he eat in 12 months? Ans. $5\frac{1}{7}$ tons.

20. If 8 square rods of ground produce 13 bushels of potatoes, how many bushels would one acre produce, at the same rate? Ans. 260.

21. If 3 pounds of sugar cost 40 cents, what must I give for 100 lbs. of the same? Ans. $13,33$\frac{1}{3}$.

22. Wishing to know the heighth of a certain steeple, I measured the shadow of the same on a horizontal plain, $27\frac{1}{2}$ feet. I then erected a ten feet pole on the same plain, and it cast a shadow of $2\frac{2}{3}$ feet. Required the height of the steeple?

Ans. $103\frac{1}{8}$. feet.

23. The earth turns upon its axis once in 23 hours and 56 minutes, and its circumference is 25000 miles. How far is a person carried in one second of time, who lives on the equator?

Ans. $92\frac{916}{1077}$ rods.

24. Two men, A and B, commence a journey at the same time and from the same place. A's speed, which is $5\frac{1}{5}$ miles an hour, bears the same proportion to B's that 5 does to 7; how far does B travel in 9 hours?

Ans. $65\frac{13}{25}$ miles.

25. A gentleman having a lot of land in the form of a parallelogram, wishes to set off 50 acres from one end which is 100 rods wide. How many rods will it require on the other side to contain 50 acres?

Ans. 80.

26. A owes B $500, but not being able to pay the whole, B offers to compound with him for a sum which shall bear the same proportion to his debt, as $\frac{2}{7}$ does to $\frac{2}{5}$; what does B receive?

Ans. 357,142+

Note. The numerator of a fraction bears the same proportion to that part of a quantity expressed by the fraction as the denominator bears to the whole quantity.

27. If a man fill $\frac{1}{3}$ of a cask with brandy, $\frac{1}{4}$ with wine, and $\frac{1}{5}$ with water, and it lack $21\frac{2}{3}$ gallons of being full, how many gallons will that cask contain?

Ans. 100.

28. If a gamester at the first game lose $\frac{1}{8}$ of his money, and win 3 shillings, at the second; lose $\frac{1}{8}$ of his first stock at the third, and win six shillings at the fourth; now if what he has won bears the same proportion to what he has lost, as what he has left of his first stock does to his first stock, how much money had he when he commenced playing?*

Ans. $52\frac{8}{9}$ shillings.

29. A and B trade until they gain 6 per cent. on their stock; then $\frac{2}{5}$ of A's gain is $18. Now if A's

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* This question is susceptible of an indefinite number of answers. The one given is the answer obtained without reducing the fraction.

stock is to B's as $\frac{2}{5}$ to $\frac{2}{6}$, what did each gain, and what was the original stock of each?

Ans. $\left\{\begin{array}{l}\text{A's gain was \$45 and his stock \$750.}\\ \text{B's gain was \$37,50 and his stock \$625.}\end{array}\right.$

30. A, B, and C united stocks in trade; A put in $160, B, $200, and C, $150; and they gain a sum equal to $\frac{1}{5}$ of their whole stock: how much does each gain?

Ans. A gains $32, B, $40, and C $30.

31. A sells B 100 lbs. of sugar at 12 cents a pound, and gains 9 per cent.; he receives of B 100 lb. of beef at 6 cents a pounds, who gains 10 per cent. Which gains the most by the trade?

Ans. A gains 44,53+ cents more than B.

32. What per cent. does a merchant gain who buys tea at 45 cents a pound, and sells it at 55?

As 45 cents are to what is gained on 45, so is 100 to what would be gained on 100, which will be the per cent. Ans. 22$\frac{2}{9}$.

33. If a man buy tea at 45 cents a pound, and sell it so as to gain 25 per cent., what does he receive per pound? Ans. 56$\frac{1}{4}$ cents.

34. B bought brandy for $1,10 per gallon, and sold it so as to lose 5$\frac{1}{2}$ cents on a dollar. What did he receive per gallon? Ans. $1,0395.

Note. The diameter of a circle bears nearly the same proportion to the circumference, that 7 does to 22.

35. If the diameter of a circle be 100 feet, what is the circumference? Ans. 314$\frac{2}{7}$ feet.

36. The diameter of the sun is 883246 miles; what is its circumference? Ans. 2'775'916 miles.

37. The diameter of Jupiter and Mercury are to each other as 44585 is to 1612. Allowing the circumference of Mercury to be 10132$\frac{4}{7}$ miles, what is the diameter of Jupiter? Ans. 89170 miles.

Note.—The spaces passed over by bodies falling freely in vacuo, are as the squares of the times. *It is found* by experiment that bodies falling in vacuo, *descend* 16$\frac{1}{16}$ feet in the first second of time.

38. If a body fall freely in a vacuo 4 seconds of time, how much farther will it descend during the fourth second than the first? Ans. $96\frac{3}{5}$ feet.

39. If a body fall $16\frac{1}{10}$ feet in the first second of its descent, how far will it have descended at the end of 5 seconds? Ans. $402\frac{1}{2}$ feet.

Note.—The squares of the periodical times of the Planets, bear the same proportion to each other as the cubes of their mean distances from the sun.

40. If two balls revolve about a common centre, —one at the distance of 3 inches, the other at the distance of 5, and the ball nearest the centre revolve once in 3 seconds; how long will the other be completing a revolution, if both observe the same laws of motion as the planets?

Ans. 6,45+seconds.

41. The Earth's mean distance from the Sun is 94'507'428 miles. Calling the periodical time of the Earth 365 days, and that of Venus $224\frac{1}{2}$, which is near the truth, what is the mean distance of Venus from the Sun? Ans. 68'351'533+miles.

42. If 15 men can do a piece of work in 14 days, in what time would 18 men do the same work, working at the same rate?

The greater number of men must bear the same proportion to the less, that the longer period of time does to the shorter. Ans. $11\frac{2}{3}$ days.

43. If $160 worth of provisions will support 80 men 12 days, how long will the same provision support 135 men? Ans. $7\frac{1}{9}$.

PROMISCUOUS QUESTIONS.

1. A man sold one hog, weighing 250 pounds, at 4 cents per pound; a second, weighing 300, at $4\frac{1}{2}$ cents; and a third, weighing 369, at 5 cents per *pound. What was the average price per pound f* *the whole?*

Explanation.—The three hogs came to $41,95; therefore, $41,95 divided by the whole weight, will give the average price per pound.

Ans. $4\frac{512}{818}$ cents.

2. If 50 gallons of brandy, worth 6 shillings per gallon; 45 gallons worth $1,05 per gallon, be mixed with 38 gallons, worth 96 cents per gallon, what will be the value of one gallon of the mixture?

Ans. $100\frac{73}{133}$ cents.

3. A and B agreed to settle a disputable account on the average judgment of three men, who all declared a balance in favour of B. C's judgment was $14, D's $15,96, E's $12,34. Agreeably to the decision, what aught A to pay B? Ans. $14,10.

4. P wishing to build a brick house, agreed with R to furnish brick for $233,334. The house was 40 feet long, 35 feet wide, and the walls were 18 feet high and 1 foot thick. There were 30 windows, each 5 ft. 6 in. high, and 2 ft. 10 in. wide. Making no allowance for doors, what did R receive per thousand for his brick.

Brick are generally made 8 *inches long,* 4 *inches wide, and* 2 *inches thick.* Ans. $4.

5. If three clerks can copy 180 pages in two days, how many pages can 7 clerks copy in 6 days, working at the same rate?

First find how many pages one will copy in one day, then how many pages he will copy in 6 *days.*

Ans. 1260.

6. If 200 dollars in 14 months gain 14 dollars interest, how much interest will 300 dollars gain in 13 months, at the same rate? Ans. $19.50.

7. If 6 men can perform a piece of work in 58 days, in how many days can 10 men do $\frac{4}{5}$ of the same work, working at the same rate?

Ans. $27\frac{21}{25}$.

8. If $594 will pay the board of 11 men 9 weeks, how many men can be boarded 12 weeks, at the same rate per week, for $1080? Ans. 15.

9. A man bought 300 pairs of shoes for $225, and exchanged 180 pairs for 150 hats. He then gave 100 hats for 90 yards of cloth, and sold the cloth for $108. What did he gain per cent.?

Ans. 20.

10. A gentleman bought 95 yards of cloth, $\frac{3}{4}$ of a yard wide, for 100 dollars, and gave the same and $25 for cloth of the same quality, one yard wide. How many yards did he buy? Ans. 89$\frac{1}{16}$.

11. The distance of West Point from the city of New-York, is 60 miles. A performed a journey on foot from the latter to the former place in 12 hours. After travelling 4 hours, he inquired the remaining distance, and was answered, that the distance he had already travelled, equalled $\frac{3}{7}$ of the remaining distance. How far had he travelled, and at what rate did he travel per mile before and after making the inquiry.

Explanation.—If the distance he had travelled be divided into parts equal to those into which the remaining distance is supposed divided, that is, so that each part should equal $\frac{1}{7}$ the remaining distance, there would be 3 parts, for the distance travelled is equal to $\frac{3}{7}$ the remaining distance, and those 3 parts added to the 7 parts remaining, gives 10, the number of parts into which the whole is divided. A had travelled over 3 of these parts, consequently he had travelled $\frac{3}{10}$ of the whole distance.

Ans. He had travelled 18 miles. His average speed before the inquiry was 4$\frac{1}{2}$ miles per hour, and after 5$\frac{1}{4}$.

Note—Answers to all questions similar to the last, may be found by making the sum of the given numerator and denominator, the denominator, and preserving the given numerator.

12. A certain philosopher being asked at what time he arose in the morning, answered, that the time he usually spent in bed after midnight, was equal to $\frac{3}{5}$ of the time intervening between the time of his rising and twelve o'clock at noon. Required the time. Ans. At half past 4 o'clock.

13. A certain clerk receives for his services $800 a year. His expenses equal $\frac{5}{11}$ of what he saves. How much of his salary does he save yearly? Ans. $550.

14. R and S received the same income yearly. R spent $\frac{3}{4}$ of his, S spent $40 more than R, which involved him in debt $10 a year. What was their yearly income?

Explanation.—Had S spent but $30 more than R, he would have just spent his income, for $40 is $10 more than one-fourth of R's income. Ans. $120.

For working questions similar to the last, see rule page 175.

15. F and H traded upon equal capitals, F gained a sum equal to $\frac{3}{8}$ of his capital,—and H a sum equal to $\frac{1}{4}\frac{1}{6}$ of his which was $500 less than F's gain. What was the capital of each? Ans. $4000.

16. A can earn 6 shillings a day. A and B can earn twice as much as C; but A and C can earn 3 times as much as B. What can B and C earn respectively?

Explanation.—The labour of C equals $\frac{1}{3}$ the labour of the three, and the labour of B equals $\frac{1}{4}$ of the labour of the three, both of which will perform $\frac{7}{12}$ of the whole; therefore 6 shillings divided by $\frac{5}{12}$, the part that A can perform, will express what the three can earn. Ans. B can earn $3\frac{3}{5}$ s. and C, $4\frac{4}{5}$ s.

17. Divide 144 into two such parts, that when the less is multiplied by 9 and the greater by 7, their products may be equal. Ans. 63 and 81.

18. A merchant paid $$293\frac{1}{3}$ for a quantity of rum, brandy and whisky,—giving 8 shillings a gallon for brandy, 6 for rum, and 3 for whisky. The brandy equalled $\frac{1}{3}$ the quantity of rum, and the rum $\frac{1}{2}$ the quantity of whisky. How many gallons of each kind did he purchase?

Ans: Brandy 40 gal. rum, 120, and whisky, 240.

19. A gave $350 for two pieces of cloth,—one

measuring 24 yards, and the other 31. The price per yard of the smaller, was to that of the larger piece as 5 to $7\frac{1}{2}$. What did he give per yard for each piece? Ans. \$$4\frac{136}{141}$ for the less, and \$$7\frac{21}{47}$ for the other.

MISCELLANY.

TAXES.

A tax is money assessed on the polls and properties of individuals for the purpose of supporting government, schools, highways, &c.

In general, the tax on each poll is fixed at a certain rate, which is first taken out of the whole tax, and the remaining part of the tax is assessed upon the property of the town, parish, district, &c. An inventory is taken of all the taxable property, both real and personal of the whole town, parish, or district; also of each individual who is to be taxed. Then each individual is taxed in proportion to his property. After knowing the amount of the inventory and the whole tax, the most convenient method of finding the tax of each individual, is to write in a table the tax upon any number of cents from 1 to 10; for 20, 30, 40, &c. to 100 cents; also the tax upon any number of dollars from 1 to 20, or a greater number if necessary. By knowing the property of any individual, it will be easy to ascertain his proportion of the tax by adding together those sums standing against his amount of property in the table. If the tax on each poll be a fixed sum, that sum should be multiplied by the whole number of polls, and the product subtracted from the whole tax as in the following example.

Suppose a tax of \$5138,20 be assessed upon a town, the inventory of which is \$25691; and that there are 209 polls, which pay 25 cents each. What will be the tax on one dollar of the inventory?

The tax on the 209 polls is $52,25, which taken from the whole tax $5138,20, leaves $5085,95 to be assessed upon the property. The remaining part of the tax must be divided into as many equal parts as there are dollars in the inventory, to ascertain what must be paid upon one dollar of the inventory. This we shall do by dividing 508595 cents by 25691, the number of dollars contained in the inventory.

Ans. The tax on one dollar will be 19,7+cents.

If the tax on each poll be established at a certain rate, the amount of the polls is considered a part of the inventory, and will pay a tax equal to the same amount of the inventory of the property, consequently their amount must be added to the inventory of the property.

Suppose the inventory of the property in a certain town, to be $41265; the number of polls 360; and that each poll is rated by law at $1,20. If a tax of $8339,40 be assessed on this town, what will be G's tax, who pays for 2 polls, and whose property is valued at $46,27?

Explanation.—360 the number of polls, multiplied by $1,20, must be added to the inventory of the property. Then $8339,40 divided by $41697, gives 20 cents the tax on one dollar; and 20 cents divided by 1 dollar, gives two mills the tax on one cent.

To form a table, we multiply 2 mills by 2, 3, 4, &c. cents, and 20 cents, by 2, 3, &c. dollars.

TABLE.

Cts.		Mills.	$		$	Cts.
1	pays	2	1	pays		20
2	"	4	2	"		40
3	"	6	3	"		60
4	"	8	4	"		80
5	"	10	5	"	1	00
6	"	12	6	"	1	20
7	"	14	7	"	1	40
8	"	16	8	"	1	60
9	"	18	9	"	1	80
10	"	20	10	"	2	00
20	"	40	20	"	4	00
30	"	60	30	"	6	00
40	"	80	40	"	8	00
50	"	100	50	"	10	00
60	"	120	60	"	12	00
70	"	140	70	"	14	00
80	"	160	80	"	16	00
90	"	180	90	"	18	00
			100	"	20	00

G's tax may now be obtained from the table, where it is found,

	that $40	pays	$8
	8	"	1,60
	,60	"	,12
	,07	"	,014
Value of G's inventory,	$48,67	"	$9,734 G's tax,

* * *

MEASURING SURFACES AND SOLIDS.

To find the superficial contents of surfaces which are not rectangles.

1. How many square feet in a board 18 feet long, 20 inches wide at one end, and 10 at the other?

Explanation.—It is evident from plate, page 23 that the average width of the board will be the width of the middle. It may also be proved by the same plate, that half the sum of the ends will be the width in the middle. Ans. 22½.

2. How many square feet in six boards, each 20 feet long, 15 inches wide, at one end, 3 inches at the other? Ans. 90.

3. How much land in a field 49 rods long, 35 rods wide at one end, 25 at the other, if the angles at the extremities of the first mentioned side, be right ones, and all the sides be right lines?

Ans. 9 A. 30 rods.

It has already been shown, that multiplying the length of a rectangle by the breadth, gives the area. The surface of a right angled triangle is equal to one half the surface of a rectangle, as long as the base of the triangle, and as wide as the perpendicular; therefore multiplying the base of any surface in the form of a right angled triangle by half the length of the perpendicular, gives the area.

4. How much land in a field in the form of a right angled triangle, if the base be 49½ rods, and the perpendicular 23 rods?

When parts of a rod are given in a vulgar fraction, or feet, it is the better way to change them to a decimal fraction. Ans. 3 A. 2 R. 9¼ rods.

5. How many feet of boards will be required to board the gable ends of a building, 40 feet wide, if a perpendicular from the beam to the ridge be 9 feet, 7 inches? Ans. 383⅓.

Note.—The contents of any irregular surface bounded by right lines, may be found by dividing it into right angled triangles, and adding the contents of the triangles so formed.

To find the contents of circles.

RULE.

Multiply half the circumference by half the diameter.

6. If the diameter of a circle be 8 feet, what is the area? Ans. $50\frac{2}{7}$ feet.

7. How many square feet on the surface of the end of a cylinder, whose circumference is 10 feet? Ans. $7\frac{21}{22}$.

8. How many rods of land will it require to make a walk one rod in width, round a circular pond, whose diameter is 10 rods? Ans. $34\frac{4}{7}$.

To find the contents of timber, or any other solid body, when the same bigness is preserved throughout the length of the same.

RULE.

Multiply the number of square inches, feet, &c. on the surface of one end, by the length.

This rule is founded on the principle illustrated, page 97.

9. How many solid feet in a stick of timber 25 feet long, $2\frac{1}{2}$ feet wide, and 1 ft. 9 in. thick? Ans. 109,375.

10. How many solid feet in a round stick of timber, $16\frac{1}{2}$ feet long, and $2\frac{1}{2}$ in diameter? Ans. $81\frac{3}{112}$.

11. What will a round stick of timber come to, at $5 per ton, if it be 20 feet long, and 7 feet in circumference? Ans. $9,73+.

12. How many solid feet in a stone, 14 feet long, having the ends in the form of a right angled triangle, the widest side being 15 inches, and one of the others 12? Also, what will the hewing come to at $4\frac{1}{2}$ cents per square foot?

Ans. $5\frac{1}{4}$ solid feet; and cost of hewing $1,92+.

13. H employed T to dig and stone a well, 30 feet deep, and 9 feet diameter; and was to give him 4 cents for each solid foot of stone, it should require in stoning the same. The quantity of stone was estimated at $\frac{3}{4}$ of the whole space except the tunnel, which was $3\frac{1}{2}$ feet diameter. How much did T receive? Ans. $48,61$\frac{17}{28}$.

To find the contents of solids larger at one end than the other, and when the size regularly decreases.

RULE.

If the stick decrease with a true slant till it terminates in a point, multiply the surface of the base, or larger end, by one third of the length. But if the body do not taper to a point multiply the surface of the larger end by $\frac{1}{3}$ the length the body would be if it continued to a point, and subtract the contents of the supposed part from the whole contents. The contents of the supposed part is found by multiplying the surface of the smaller end of the given part, into one-third of the length of the supposed part.

Note When the body does not taper to a point, the length it would be if it did, may be found by the following proportion.

As the difference between the smaller diameter, or thickness, and the larger diameter, or thickness, is to the length of the given stick, so is the diameter, or thickness, of the larger end to the length required. The given length taken from the whole, will leave the length of the supposed part.

14. What is the contents of a round stick of timber, 15 feet long, the surface of one end being 2$\frac{1}{2}$ feet, and the other a point?

Ans. 12$\frac{1}{2}$ feet.

15. How many solid feet in a stick of timber, whose base is 20 inches square, and its length 60 feet, if it decrease to a point?

Ans. 55$\frac{5}{9}$.

16. How much timber in a stick, 23 feet long if the larger end be two feet square, and the other 1,5 feet? Ans. 1 T. 20$\frac{1}{12}$ feet.

A more concise method of finding the contents of a frustum of a cone or pyramid, that is, of bodies tapering with a true slant, when they do not come to a point, may be found in the following rule.

RULE 2.

Add together the areas of each end and their mean proportional, and multiply their sum by $\frac{1}{3}$ of the length.

17. Required the contents of a round stick of timber, $17\frac{1}{2}$ ft. long, the diameter of the larger end being $2\frac{1}{2}$ feet, and that of the smaller end, 1 foot?

Ans. $44\frac{1}{16}$ feet.

18. If a stick of timber, 60 feet long, and $1\frac{1}{2}$ feet square at the large end, tapering to a point at the other, be sawed off in the middle, how many more solid feet will there be in the larger part than in the smaller. Ans. 33,75.

19. A gentleman purchased two stone pillars for 3,25 dollars per solid foot. The base of each was 2 feet in diameter, the other end $1\frac{1}{4}$ feet, and the length 12 feet. What was the cost of both pillars?

Ans. $164,645+.

EXCHANGE.

The value of the pound, shilling, pence, and farthing, varies in different parts of the United States. The value is determined by the number of shillings in a dollar, as the value of a dollar is the same in all the States.

The following table exhibits the number of shillings in each state, of which the dollar is composed.

TABLE.

	s.			Dollar.
In New-England,	6		=	1
" Virginia,	6		=	1
" Kentucky,	6		=	1
" Tennessee,	6		=	1
" New-York,	8		=	1
" North-Carolina,	8		=	1
		d		
" Pennsylvania,	7,	6	=	1
" New-Jersey,	7,	6	=	1
" Delaware,	7,	6	=	1
" Maryland,	7,	6	=	1
" Ohio,	7,	6	=	1
" South-Carolina,	4,	8	=	1
" Georgia,	4,	8	=	1

Any number of pounds, shillings, pence, and farthings, in the currency of any state, may be changed to Federal Money, by reducing all the less denominations to the decimal of a pound, and dividing the whole by a fraction, made by taking the number of shillings which equal a dollar for a numerator, and 20 the number of shillings in a pound, for a denominator.

For reducing shillings, pence, and farthings to the decimal of a pound, the student is referred to Expl. of question 52, page 152,—to rule, page 182 and to the several Expls. under that rule.

The reasons of the foregoing observations, will at once appear to the learner, when he considers that the fraction will be the same part of a pound, that one dollar is, and every time the fraction is taken from the number of pounds given, an unit is put in the quotient, which must be one dollar.

Note.—If the value of a dollar be expressed in shillings and pence, reduce the shillings to pence and add the given pence for a numerator, and 20 shillings to pence for a denominator.

1. In 24 pounds New-England currency, how many dollars?

Explanation.—6 s. equal $\frac{6}{20}$ or $\frac{3}{10}$ of a pound, therefore there will be as many dollars in 24 pounds as $\frac{3}{10}$ can be taken times from 24.

Ans. $80.

2. How many dollars in 21 £. 12 s. New-England currency? Ans. 72.

3. Change 9 £. 10 s. 6 d. New-England currency to Federal money. Ans. $31,75.

4. Change 5 s. 3½ d. New-England currency to Federal money. Ans. $0,881+.

5. In 15 £. 15 s. New-York currency, how many dollars? Ans. $39,375.

6. Change 17 s. 10 d. 3 qr. New-York currency, to Federal money. Ans. $2,236+.

7. In 18 £. 17 s. 4 d. 1 qr. Pennsylvania currency, how many dollars? Ans. $50,313+.

8. Reduce 49 £. 7 d. Maryland currency, to Federal money. Ans. $130,744+.

9. Change 10 £. 3 qr. South-Carolina currency, to Federal Money. Ans. $42,87+.

10. In 19 s. 11 d. Georgia currency, how many dollars? Ans. $4,267+.

But few countries make use of the same coin, or coins of the same value.

The following table shows the value of several foreign coins in dollars and cents.

TABLE.

1 Pound sterling of Great Britain	=$4,44.
1 Pound sterling of Ireland	= 4,10.
1 Pagoda of India	= 1,84.
1 Milrea of Portugal	= 1,24.
1 Tale of China	= 1,48.
1 Rix dollar of Sweeden and Denmark	= 1,00.
1 Silver Rouble of Russia	= ,75.
1 Rupee of Bengal	= ,50.
1 Florin or Guilder of the kingdom of the Netherlands	= ,40.
1 Mark Banco of Hamburg	= ,33$\frac{1}{3}$.
1 Frank of France	= ,18$\frac{3}{4}$.
1 Livre tournois of France	= ,18$\frac{1}{2}$.
1 Real of Plate of Spain	= ,10.
1 Real of Vellon of Spain	= ,05.

When the value of any number of foreign coin is required in Federal money, if that number be multiplied by the value of one in dollars and cents, the product will express the value of the whole in dollars and cents, for the product contains the multiplier as many times as the multiplicand contains an unit.

1. A merchant bought 45 crates of ware in Liverpool for 245 pounds sterling; how many dollars did he give for each crate? Ans. $24,173+.

2. A merchant bought 50 tons of iron in St. Petersburg for 40 roubles per ton. He paid for importation and duties $2000. What did the iron cost him per ton? Ans. $70.

3. In 76 pounds 13 shillings of Irish currency, how many dollars? Ans. $314,265.

4. Bought 2361 flasks of olive oil at Bordeaux for 787 franks, and paid other charges to the amount of $120. The whole was sold in Philadelphia for $377,76. What was the gain on one flask?

Ans. 4 cents, 6+mills.

5. A sea-captain obtained board in Hamburgh 6 weeks for 112 mark bancos. What did his board cost him per day in Federal money? Ans. $0,888+.

6. If a man pay 10 tales per chest for tea in Canton, how much must he sell it for per chest to gain 35 per cent., allowing the cost of importation and duties to be 3 times the original cost? Ans. $79,92.

THE END.

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15
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